Mode-Locking of Lasers

Herman A. Haus, *Life Fellow, IEEE*

Invited Paper

***Abstract*—The evolution of the theory of mode-locking over the last three and a half decades is reviewed and some of the salient experiments are discussed in the context of the theory. The paper ends with two-cycle pulses of a mode-locked Ti : Sapphire laser.**

***Index Terms*—Mode-locked lasers, optical pulses, pulsed lasers, pulse generation, ultrafast optics.**

I. INTRODUCTION

THE ABSOLUTE bandwidth of the gain of optical lasers is large, approaching an octave in the case of Ti : Sapphire. Thus, lasers can amplify broad-band radiation. If the radiation is in the form of pulses, the pulses can be very short.

The word mode-locking describes the locking of multiple axial modes in a laser cavity. By enforcing coherence between the phases of different modes, pulsed radiation can be produced. Mode-locking is a resonant phenomenon. By a relatively weak modulation synchronous with the roundtrip time of radiation circulating in the laser, a pulse is initiated and can be made shorter on every pass through the resonator. The shortening process continues unabated, until the pulse becomes so short and its spectrum so wide that pulse lengthening mechanisms or spectrum narrowing processes spring into action, such as finite bandwidth of the gain. The history of laser mode-locking is a progression of new and better ways to generate shorter and shorter pulses, and of improvements in the understanding of the mode-locking process.

The first indications of mode-locking appear in the work of Gürs and Müller [1], [2] on ruby lasers, and Statz and Tang [3] on He–Ne lasers. The first papers clearly identifying the mechanism were written in 1964 by DiDomenico [4], Hargrove *et al.* [5], and Yariv [6]. Hargrove *et al.* achieved mode-locking by internal loss modulation inside the resonator. This is the case of "active" mode-locking. Mocker and Collins [7] showed that the saturable dye used in ruby lasers to Q-switch the laser could also be used to mode-lock. Q-switching is a process in which the laser is caused to emit pulses that are many roundtrips in duration. The saturable absorber is bleached by the radiation in the resonator. The emission of radiation stops when the gain medium is depleted, and the process starts all over again. Mocker and Collins observed that the Q-switched pulse broke up into a train of very short pulses separated by the roundtrip time. The train carried the same energy as the Q-switched pulse and hence the pulses were of much greater peak intensity than the pulses produced by Q-switching alone. This was the first example of passive mode-locking. For several years, techniques were developed for measurement of these pulses and for their use to probe nonlinear response of optical media. The measurement accuracy was impaired by the somewhat unpredictable nature of the transient mode-locking. This drawback was overcome when Ippen, Shank, and Dienes [8] generated the first CW saturable absorber mode-locking using a saturable dye in a dye laser. Shortly thereafter this led to production of pulses of sub-picosecond duration [9]. The reproducible character of these pulses improved the accuracy of pump-probe measurements by four orders of magnitude. The work on dye lasers continued unabated for the next decade producing shorter and shorter pulses [10]–[12]. Ultimately, a record 6-fs pulse duration was achieved by Fork *et al.*using pulse compression external to the cavity [13]. The pulse compression technique uses the Kerr nonlinearity of an optical medium. The pulses propagating through the medium experience nonlinear phase shifts that lead to spectral broadening. The spectral broadening results in a chirp, a spread of frequencies. The different frequency components are superimposed by propagation in a dispersive medium, or by reflection from a grating pair.

The analytic theory of active mode-locking was firmly established in a classic paper by Siegman and Kuizenga [14]. The process was studied in the frequency domain with explicit attention paid to the injection locking of the axial modes by a loss modulator. The predicted pulse shape was Gaussian. This fact was confirmed by experiment. The analytic theory of passive mode-locking had to await the advent of CW mode-locking, since transient mode-locking is too complicated to yield to an analytic approach. The author became interested in the problem upon hearing of the successful CW mode-locking of dye lasers [8]. How the pulses were formed was not clear in this case, since it was known from previous work [15] that the relaxation time of the absorber was much longer than the pulse generated in the laser. This puzzle intrigued the author and he set out to develop an analytic theory of saturable absorber mode-locking on one of his sabbaticals at the Bell Laboratories. Ignoring, at first, this puzzle, he developed an analytic theory of passive mode-locking with a fast saturable absorber, an absorber with a relaxation time short compared with the pulsewidth [16]. This required a reformulation of the mode-locking theory of Siegman

Manuscript received August 10, 2000. This work was supported by the Air Force Office of Scientific Research (AFOSR) under Grant F49620-98-1-0139.

The author is with the Department of Electrical Engineering and Computer Science and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

Publisher Item Identifier S 1077-260X(00)11589-8.

and Kuizenga [14] in the time domain. At about the same time New [17] explained by computer simulation the operation of the mode-locked dye laser: The laser shaped its pulse by the leading edge of the absorber saturation, which opens a window of net gain. The window is closed by gain saturation. These two processes produce a window of net loss decrease, just like a fast saturable absorber. The analytic theory developed by the author predicted hyperbolic secant temporal envelopes of the electromagnetic field. He approached with this information Ippen and Shank who were making pulse shape measurements at the time, and the prediction was confirmed by experiments [18].

In the meantime, the theory of mode-locking with a fast saturable absorber lay fallow, since nature does not provide us with an absorber with a relaxation time much shorter than a picosecond. Then came Mollenauer's soliton laser [19]. The soliton laser consisted of two resonators, one active the other passive and containing a fiber, coupled via a semitransparent mirror to the laser resonator. The two resonators were feedback stabilized to within a fraction of a wavelength. The operation of this laser was explained afterwards as an interference phenomenon between the pulses circulating in the two subresonators and interfering at the semitransparent mirror [20]. Through proper phasing a net pulse shaping is produced analogous to that of a fast saturable absorber. The process was dubbed additive pulse mode-locking (APM) [21]. The principle was generalized to fiber ring lasers in which the APM action is produced by a birefringent element in the resonator. Via polarization controllers the pulse is split into two co-propagating versions in the fiber. The interference of the two polarizations at the output polarizer leads to effective fast-saturable-absorber action.

Then came the surprising result of Sibbett's group at the University of St. Andrews [22] who generated very short pulses in a single resonator. This work was pursued at several laboratories [23]–[29] and the mode-locking was recognized as caused by what is now called Kerr–lens mode-locking (KLM). The effect of a fast saturable absorber is simulated by Kerr focusing: the high intensity part of the beam is focused by the Kerr–effect, whereas the low intensity parts remain unfocused. If such a beam is passed through an aperture, the low intensity parts are attenuated, thereby shortening the pulse.

Pulses can be shaped by means other than saturable absorber action. A pulse propagating in a Kerr–medium with anomalous dispersion (such as in a fiber in the wavelength regime of $\lambda > 1.3$ μm) can form into a soliton. This effect, by itself, is capable of producing pulses in a Hamiltonian (loss-free) way. The Kerr–nonlinearity is balanced by dispersion. In the presence of gain, the buildup of noise in the intervals between the pulses must be prevented by (effective) saturable absorber action. However, the pulse shaping function may be performed by soliton-shaping alone. Fiber lasers have been mode-locked in this way [30].

There is another variant of this pulse shaping by the Kerr–effect and dispersion, in a way analogous to dispersion managed soliton propagation [31]. The dispersion in the ring may be made to vary from normal to anomalous by proper splicing of fiber segments. The pulse inside the resonator stretches and compresses [32]. The net dispersion may be zero, yet soliton-like pulse shaping is still possible [31]. The reason for this is the fiber nonlinearity that causes the pulse spectrum to be narrower in the segment with normal dispersion than in the segment with anomalous dispersion. Thus, on the average, the pulse experiences anomalous dispersion which balances the Kerr–effect.

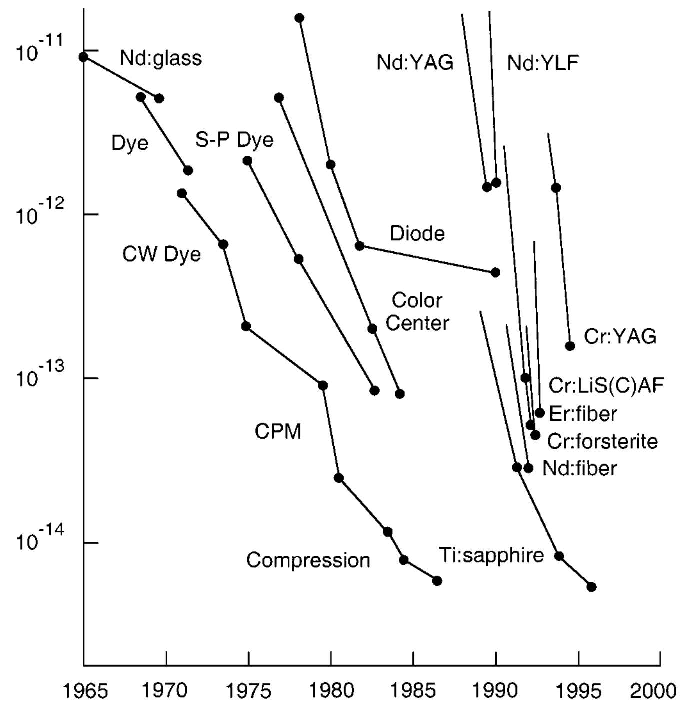

Fig. 1. Pulsewidths of different laser systems achieved year by year.

The KLM mode-locking mechanism is a very effective way of generating short powerful mode-locked pulses from Ti : Sapphire and was carried on and perfected by groups at the University of Washington [33]–[35], Vienna [36]–[39], MIT [40]–[43], [78], and ETH [44]–[48]. The Vienna group introduced chirped mirrors for dispersion compensation, the ETH group developed the double chirped mirror design and introduced saturable semiconductor mirrors to ensure self-starting of mode-locking.

The recent record of about 5 fs [49], [50], the shortest pulses ever achieved directly from an oscillator, is explained by the dispersion managed soliton model [51]. The Ti : Sapphire gain crystal has positive dispersion, the mirrors are carefully designed to balance this dispersion. When the pulses are extremely short, they are stretched and compressed by more than a factor of two in width as they propagate through the resonator.

Fig. 1 shows the history of mode-locking in terms of the years in which shorter and shorter pulses were achieved. Distinct branches pertaining to different laser types are shown. The earliest successful generation of sub-picosecond pulses was achieved with dye-lasers. Then came the mode-locking of semiconductor lasers. Finally, the shortest pulses were generated with solid state systems, in particular with Ti : Sapphire.

This paper is a brief summary of the history of mode-locking, from its beginning in the 1960s. The literature is enormous. In the excellent review chapter of early work written in 1974 [52] already over 400 references are cited. Today, a complete bibliography would run into many thousands of references. Instead of providing such a list, I believe that the reader will be better served if presented by a concise review of the main theoretical concepts and equations that describe mode-locking. The theoretical work on mode-locking extends over more than 35 years. Notation has changed with the times. In this paper, we shall describe the most important models of mode-locking in one common notation. We start in Section II with a description of active mode-locking in the frequency domain and the time domain. We develop the master equation and find the Gaussian pulse solutions. In Section III we present the master equation for fast saturable absorber mode-locking and find its hyperbolic secant solutions. In Section IV we briefly review mode-locking with a slow saturable absorber. Section V looks at APM and KLM. Section VI introduces the Kerr–effect and group velocity dispersion into the master equation. Section VII treats the stretched pulse fiber laser. In Section VIII we look at the latest results in ultrashort mode-locked pulse generation.

II. Active Mode-Locking in the Frequency and Time Domains

An optical Fabry–Pérot resonator formed of two mirrors has axial modes separated in frequency by $\Delta\Omega = 2\pi/T_R$, where T_R is the roundtrip time. A laser is formed by introducing a gain medium. Generally, several axial modes will be lasing if the gain level is above threshold. Denote the frequency of the central mode by ω_o. The laser is mode-locked by an amplitude modulator placed near one of the mirrors (see Fig. 2). A cosinusoidal modulation of the central mode at the frequency $\Omega_M = \Delta\Omega$ produces sidebands at $\omega_o \pm \Delta\Omega$. These injection lock the adjacent modes, which in turn lock their neighbors. Denote the amplitude of the axial mode of frequency $\omega_o + n\Delta\Omega$ by A_n. The amplitude changes within each pass through the amplifier of loss $1-\ell$ and peak gain $1+g$, where $\ell \ll 1$, $g \ll 1$

$$\Delta A_n = \left\{ \frac{g}{1+\left(\frac{n\Delta\Omega}{\Omega_g}\right)^2} - \ell \right\} A_n + \frac{1}{2} M(A_{n-1} - 2A_n + A_{n+1}) \tag{1}$$

where M is the modulation. This expression can be transformed into a standard operator by introducing three approximations.

1) The frequency dependent gain can be expanded to second order in $n\Delta\Omega$.
2) The discrete frequency spectrum with Fourier components at $n\Delta\Omega$ is replaced by a continuum spectrum, a function of $\Omega = n\Delta\Omega$.
3) The sum $(A_{n+1} - 2A_n + A_{n-1})/\Delta\Omega^2$ can be replaced by a second derivative with respect to frequency if the spectrum is very dense (usually thousands of modes are involved in mode-locking).

Equation (1) becomes

$$\Delta A(\Omega) = (g-\ell)A(\Omega) - g\left(\frac{\Omega}{\Omega_g}\right)^2 A(\Omega) + \frac{1}{2} M\Omega_m^2 \frac{d^2 A}{d\Omega^2} \tag{2}$$

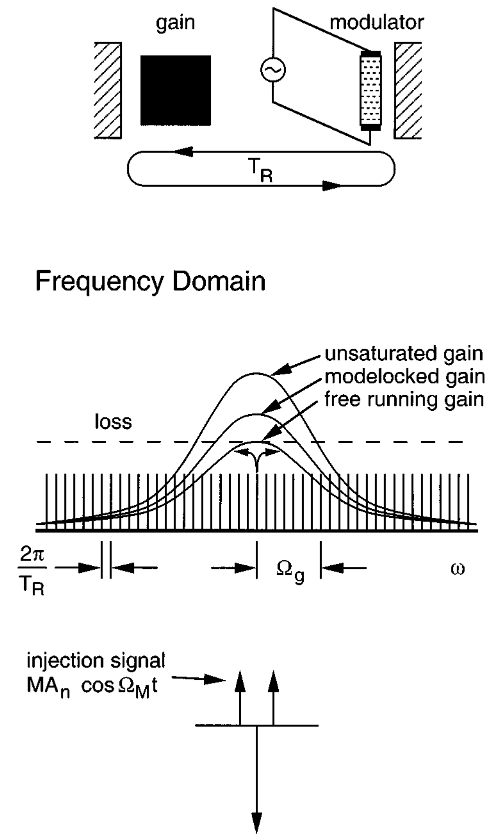

Fig. 2. Schematic of actively mode-locked laser, the spectrum, and the injections signal produced by modulator.

where $\Omega_m = \Delta\Omega$ is the modulation frequency. In the steady-state, the change of the pulse in one roundtrip is zero. Hence, the mode-locked pulse must be a solution of the differential equation

$$(g-\ell)A(\Omega) - g\left(\frac{\Omega}{\Omega_g}\right)^2 A(\Omega) + \frac{1}{2} M\Omega_m^2 \frac{d^2 A}{d\Omega^2} = 0. \tag{3}$$

The solution is a Gaussian pulse

$$A(\Omega) = A_o \exp(-\Omega^2\tau^2) \tag{4}$$

where

$$\tau^4 = \frac{2g}{\left(M\Omega_m^2 \Omega_g^2\right)}. \tag{5}$$

This is the Kuizenga–Siegman formula [14] for the pulsewidth, which is proportional to the inverse of the geometric mean of the gain bandwidth and the modulation frequency. The eigenvalue of the equation gives an expression for the net gain

$$g - \ell = \tfrac{1}{2} M\Omega_m^2 \tau^2. \tag{6}$$

The gain is greater than the loss. This is permissible, and does not cause instabilities, since the modulator increases the loss in the pulse wings.

Equation (2) can be generalized to include transient buildup or decay of the excitation. The change per pass need not be zero. If nonzero, a difference equation results for $A_N(\Omega)$, the spectrum on the Nth pass. If the evolution is slow, the difference equation can be replaced by a differential equation in terms of the long term time variable T. Even though the pulse is characterized in terms of its spectrum, slow variation of the spectrum is legitimately described in terms of a time varying spectrum. One obtains the two-dimensional differential equation

$$\frac{1}{T_R}\frac{\partial}{\partial T}A(T,\Omega)=(g-\ell)A(T,\Omega)-g\left(\frac{\Omega}{\Omega_g}\right)^2 A(T,\Omega)+\frac{1}{2}M\Omega_m^2\frac{\partial^2 A(T,\Omega)}{\partial\Omega^2}. \tag{7}$$

The transient behavior is easily characterized by an expansion of the excitation in terms of Hermite–Gaussians. One may write

$$A(T,\Omega)=\sum_n C_n(T)H_n(\Omega\tau)\exp(-\Omega^2\tau^2/2). \tag{8}$$

One obtains total differential equations for the amplitudes $C_n(T)$

$$\begin{aligned}\frac{dC_n}{dT}&=\left[g-\ell-\left(n+\frac{1}{2}\right)M\Omega_m^2\tau^2\right]C_n\\&=-nM\Omega_m^2\tau^2.\end{aligned} \tag{9}$$

Higher order excitations all experience net loss. The Hermite–Gaussians form a complete set of functions. Therefore the analysis is complete. It provides a clear picture of the behavior of pulse perturbations, Hermite–Gaussians of order $n \geq 1$. The dynamics of amplitude perturbations of the pulse itself represented by the $n=0$ term of the Hermite–Gaussian expansion require separate study. Such a perturbation, when in phase with the main pulse, lowers the gain through gain saturation (generally with a relaxation time much longer than the pulse). This provides net loss to the perturbation which then decays. Conversely, a perturbation in antiphase raises the gain and thus the original pulse amplitude is restored. This interplay between gain and resonator response can also lead to damped relaxation oscillations of the pulse-train envelope, a phenomenon we shall not discuss any further. A perturbation $n=0$ in quadrature with the pulse does not affect the energy to first order and does not affect the gain. Its decay rate is zero. Hence it is not stabilized. Amplified spontaneous emission noise kicks the phase back and forth. The phase experiences a random walk.

The description of mode-locking in terms of a pulse spectrum that evolves with time can be transformed into a description of a pulse with a temporal envelope that evolves on a time scale much longer than the pulsewidth. This is accomplished by a Fourier–transform with the Fourier–transform pairs

$$a(t)=\int d\Omega\,\exp(j\Omega t)A(\Omega) \tag{10}$$

$$A(\Omega)=\frac{1}{2\pi}\int dt\,\exp(-j\Omega t)a(t). \tag{11}$$

The pulse evolution equation becomes

$$\frac{1}{T_R}\frac{\partial}{\partial T}a(T,t)=(g-\ell)a(T,t)+g\left(\frac{1}{\Omega_g}\right)^2\frac{\partial^2}{\partial t^2}a(T,t)-\frac{1}{2}M\Omega_m^2 t^2 a(T,t). \tag{12}$$

The same master equation could have been obtained by starting with a pulse-shape expressed in time, modulated by the modulation function $M\cos(\Omega_m t)$ and expanding the modulation function to second order in t. The effect of the gain filtering is expressed by a second derivative in time.

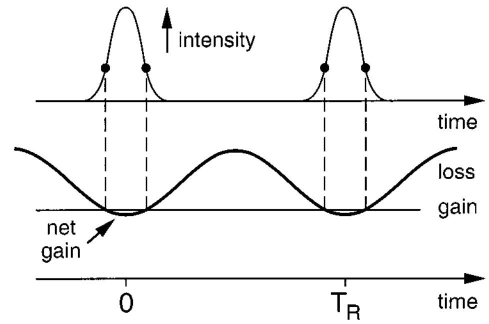

Fig. 3. Actively mode-locked pulse in the time domain and the time dependence of net gain.

The evolution of the pulse is written in terms of a long-term time variable T. Clearly, the eigenfunctions of the right hand side of (12) are Hermite–Gaussians, the consequence of the fact that the Fourier transforms of Hermite–Gaussians are themselves Hermite–Gaussians. The pulse shape of steady state mode-locking is $A_o\exp(-t^2/2\tau^2)$. In all other respects, the analysis carries through in the same way as with the Fourier transform. Fig. 3 shows the active mode-locking process in the time domain. The modulation provides a time dependent loss. Whenever the loss dips below the gain level, the curvature of the envelope is negative. The transition beween net gain and net loss marks the point of inversion on the pulse envelope.

Active modelocking does not lead to ultrashort pulses, because the frequency of modulation cannot be raised arbitrarily. Harmonic mode-locking allows for modulation frequencies at a harmonic of $2\pi/T_R$. When this is done the shortening of the pulse within each pass can be enhanced. However, if the energy of the individual pulses is to be kept high only one pulse must be allowed to circulate in the resonator. This can be accomplished through the use of step-recovery diodes for the modulation source. However, the bandwidth of optical modulators is limited and thus modulation with ultrashort electrical pulses runs into difficulties. Modulation by a passive, saturable absorber is much more effective in pulse shaping. Since the pulse itself produces the shape of the modulation function, considerably tighter modulation becomes feasible. Each time the pulse passes through the resonator it is multiplied by a time function. If the process is treated in the frequency domain, the multiplication in the time domain becomes convolution in the frequency domain. A master equation results that involves convolution integrals. For this reason, a description of passive mode-locking is best carried out in the time domain.

III. Passive Fast Saturable Absorber Mode-Locking

Passive mode-locking with a fast saturable absorber is much more easily described than mode-locking with a slow absorber. Analytic solutions are obtained with a very simple approximation. Even though no practical passively mode-locked laser existed that fitted the model of a fast saturable absorber at the time I published this theory [16], it is a coincidence that the more recent schemes of passive mode-locking are well described by an extension of this model, as we shall see later on.

In passive mode-locking, the modulator is replaced by a saturable absorber as shown in Fig. 4. The (loss) modulation of

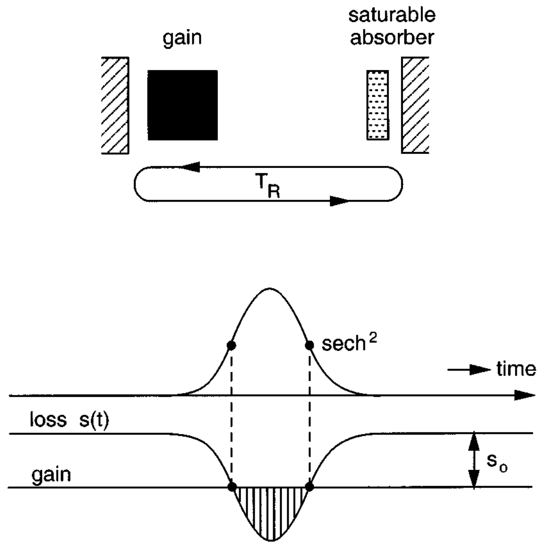

Fig. 4. Schematic of laser passively mode-locked with fast saturable absorber and the time dependence of pulse, and net gain.

the saturable absorber $s(t)$ in transmission through the absorber through the saturable absorber is

$$s(t) = \frac{s_o}{1 + I(t)/I_{\text{sat}}} \tag{13}$$

where

s_o (<1) unsaturated loss;
$I(t)$ dependent intensity;
I_{sat} saturation intensity of the absorber.

If the saturation is relatively weak, expression (13) can be expanded to give

$$s(t) = s_o - s_o I(t)/I_{\text{sat}}. \tag{14}$$

The intensity multiplied by the effective area of the mode A_{eff} gives the power in the mode. We normalize the mode amplitude so that $|a(t)|^2 =$ power. Then the transmission can be written

$$s(t) = s_o - \frac{s_o |a(t)|^2}{I_{\text{sat}} A_{\text{eff}}} \equiv s_o - \gamma |a(t)|^2 \tag{15}$$

nbwhere γ is the self amplitude modulation (SAM) coefficient. The master equation of passive mode-locking with a fast saturable absorber is obtained by introducing the saturable loss into (12) and omitting the active modulation term. The unsaturated loss s_o can be incorporated into the loss coefficient with the result

$$\frac{1}{T_R} \frac{\partial}{\partial T} a = (g - \ell) a + \frac{g}{\Omega_g^2} \frac{\partial^2}{\partial t^2} a + \gamma |a|^2 a. \tag{16}$$

The solution is a simple hyperbolic secant

$$a_o(t) = A_o \operatorname{sech}(t/\tau) \tag{17}$$

with

$$\frac{1}{\tau^2} = \frac{\gamma A_o^2 \Omega_g^2}{2g} \tag{18}$$

and

$$\ell - g = \frac{g}{\Omega_g^2 \tau^2}. \tag{19}$$

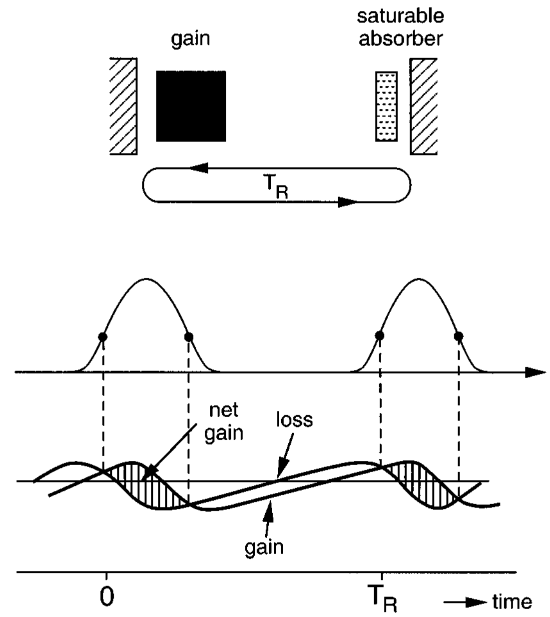

Fig. 5. Time dependent loss.

Equation (18) for the square of the inverse pulsewidth is related to that of active mode-locking, (5). Indeed, in (5) the fourth power of the inverse pulsewidth is proportional to the modulation strength M, the curvature of the modulation as a function of time Ω_m^2 and the square of the filter bandwidth. In the case of passive mode-locking, the product of modulation and curvature is equal to $\gamma A_o^2/\tau^2$. Thus (18) and (5) are indeed related. The comparison also explains why passive mode-locking can result in much shorter pulsewidths for the same filter bandwidth. As the pulse gets shorter, the curvature of the modulation increases as $1/\tau^2$, whereas it remains unchanged for active mode-locking. The pulse shape and the net temporal gain are shown in Fig. 5. The net gain is negative preceding and following the pulse. At the pulse peak, the gain is positive due to the bleaching of the saturable absorber. Another interesting feature emerges—a hyperbolic secant has exponential tails. Such tails are mandatory in passive mode-locking; the system behaves linearly in the tails since the intensity is small. The second order differential equation dictates exponential solutions for any bounded pulse.

The solution (16) is not stable unless gain saturation is explicitly included. This gain saturation can be cumulative, so that one may still make the assumption that the gain is approximately constant during the passage of one pulse. The solution is also stabilized if the full saturation behavior (13) is heeded, but then no closed form solutions have been found.

IV. Mode-Locking with a Slow Saturable Absorber

The theory of the fast saturable absorber [16] was worked out by the author in preparation for the analysis of the dye laser. The model for the slow saturable absorber has to take into account the change of gain in the passage of one pulse [17], [53]. The relaxation equation of the gain, in the limit of a pulse short compared with its relaxation time, can be approximated by

$$\frac{dg}{dt} = -g \frac{|a(t)|^2}{W_g}. \tag{20}$$

The coefficient W_g is the saturation energy of the gain. Integration of the equation gives

$$g(t) = g_i \exp - \int_o^t dt|a(t)|^2/W_g \tag{21}$$

where g_i is the initial gain before the arrival of the pulse. A similar equation holds for the loss of the saturable absorber whose response (loss) is represented by $s(t)$

$$s(t) = s_i \exp - \int_o^t dt|a(t)|^2/W_s \tag{22}$$

where W_s is the saturation energy of the saturable absorber. If the background loss is denoted by ℓ, the master equation of mode-locking becomes

$$\begin{aligned}\frac{1}{T_R}\frac{\partial}{\partial T}a = \Bigg[& g_i\left(\exp - \int_o^t dt|a|^2/W_g\right) - \ell \\ & - s_i \exp\left(-\int_o^t dt|a|^2/W_s\right)\Bigg] a \\ & + \left(\frac{1}{\Omega_f}\right)^2 \frac{\partial^2}{\partial t^2} a.\end{aligned} \tag{23}$$

Here we have expressed the filtering action as produced by a separate fixed filter, rather than by the finite bandwith of the gain (which varies with time) so as to obtain analytic solutions of the master equation. An analytic solution to this integro-differential equation can be obtained with one approximation: the exponentials are expanded to second order. This is legitimate if the population depletions of the gain and saturable absorber media are not excessive. Consider one of these expansions

$$\begin{aligned} & s_i \exp\left(-\int_o^t dt|a|^2/W_s\right) \\ & \approx s_i\left\{1 - \left(\int_o^t dt|a|^2/W_s\right) + \frac{1}{2}\left(\int_o^t dt|a|^2/W_s\right)^2\right\}.\end{aligned} \tag{24}$$

Suppose the pulse is a symmetric function of time. Then the first power of the integral gives an antisymmetric function of time, its square is symmetric. An antisymmetric function acting on the pulse $a(t)$ causes a displacement. Hence, the steady state solution does not yield zero for the change per pass, the derivative $(1/T_R)(\partial a/\partial T)$ must be equated to a time shift Δt of the pulse. When this is done one can confirm easily that $a(t) = A_o \operatorname{sech}(t/\tau)$ is a solution of (24) with the constraints on the coefficients:

$$\begin{aligned}\frac{1}{\Omega_f^2\tau^2} + g_i\left[1 - \frac{A_o^2\tau}{W_g} + \left(\frac{A_o^2\tau}{W_g}\right)^2\right] - \ell & \\ - s_i\left[1 - \frac{A_o^2\tau}{W_s} + \left(\frac{A_o^2\tau}{W_s}\right)^2\right] = 0 &\end{aligned} \tag{25}$$

$$\frac{\Delta t}{\tau} = g_i\left[\frac{A_o^2\tau}{W_g} - \left(\frac{A_o^2\tau}{W_g}\right)^2\right] - s_i\left[\frac{A_o^2\tau}{W_s} - \left(\frac{A_o^2\tau}{W_s}\right)^2\right] \tag{26}$$

$$\frac{1}{\tau^4} = \frac{\Omega_f^2 A_o^4}{4}\left(\frac{s_i}{W_s^2} - \frac{g_i}{W_g^2}\right). \tag{27}$$

These equations have important implications. Consider first the equation for the inverse pulsewidth, (27). In order to get a real solution, the right hand side has to be positive. This implies that $s_i/W_s > g_i/W_g$. The saturable absorber must saturate more strongly than the gain medium in order to open a net window of gain (see Fig. 5). This was accomplished in a dye laser system by stronger focusing into the saturable absorber-dye jet than into the gain-dye jet. Equation (25) makes a statement about the net gain before passage of the pulse. The net gain before passage of the pulse is

$$\begin{aligned}g_i - s_i - \ell = -\frac{1}{\Omega_f^2\tau^2} + g_i\left[\frac{A_o^2\tau}{W_g} - \left(\frac{A_o^2\tau}{W_g}\right)^2\right] & \\ - s_i\left[\frac{A_o^2\tau}{W_s} - \left(\frac{A_o^2\tau}{W_s}\right)^2\right]. &\end{aligned} \tag{28}$$

This gain is negative since the effect of the saturable absorber is larger than that of the gain. Since the pulse has the same exponential tail after passage as before, one concludes that the net gain after passage of the pulse is the same as before passage and thus also negative. The pulse is stable against noise buildup both in its front and its back.

The preceding analysis was carried out while the author spent a sabbatical at Bell Laboratories in 1974–1975. The published paper contains the derivation reproduced here, with a new notation that unifies the mode-locking work performed by the author and his colleagues over the next two decades. While working at the Bell Laboratories, the author concluded that integrated optics was held up in its development because no compact source of pulses for digital communications existed at the time. The processes in semiconductor lasers are analogous to those of dye lasers. A p-n junction above threshold produces gain with relaxation times of the order of 100 ps. Below threshold it provides saturable absorption with roughly the same relaxation time. Therefore, one could transfer the principle of mode-locking of dye-lasers to semiconductor lasers consisting of two junctions, driven by currents above and below threshold. He started mode-locking experiments in his group at MIT which succeeded in generating an actively mode-locked pulse train of 18 ps pulses [54]. Ippen *et al.* at Bell Laboratories subsequently achieved the first passive modelocking of a semiconductor laser [55]. Integration of the saturable absorber section with the lasing section is now a well established technology.

V. Additive Pulse- and Kerr–Lens Mode-Locking

Artificial fast saturable absorbers are produced by coherent superposition at a beam splitter or polarizer of two versions of the same pulse, one version of which is passed through a Kerr medium. The earliest realization accomplished this with two coupled resonators [20] and prompted the description APM [21]. Fig. 6 shows a single-armed realization with polarization transformers and polarizers. Linearly polarized light is transformed into elliptically polarized light which is then passed through an isotropic Kerr–medium. Elliptic polarization is rotated in the Kerr–medium by an intensity dependent angle. If the output light is again linearly polarized by an analyzer, the throughput of the system is intensity dependent. An artificial saturable absorber has been realized. This kind of APM is particularly convenient for fiber lasers, since rabbit-ear polarization transformers and polarizers are all fiber-compatible.

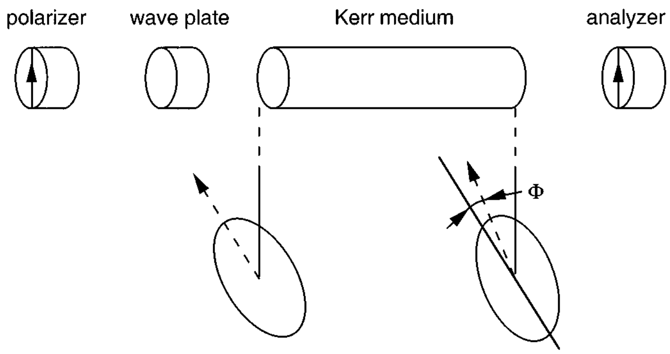

Fig. 6. Construction of saturable absorber using rotation of elliptic polarization in isotropic medium.

The rotation of elliptical polarization in an isotropic Kerr medium can be understood best by writing the Kerr–polarization in terms of circular polarization components. If the medium responds instantaneously, the polarization is [56]

$$P_{\pm} = \epsilon_o \frac{\chi^{(3)}}{2} (|E_{\pm}|^2 + 2|E_{\mp}|^2) E_{\pm}. \tag{29}$$

The isotropic character of the medium dictates that a term of the form $E_{\mp}^2 E_{\pm}^*$ does not appear in the response. Indeed, if such a term appeared, the response would be sensitive to the relative phase between the electric field E_+ and E_-. But the relative phase determines the orientation of the E-field in the special case of a linear polarization with $|E_+| = |E_-|$ and the Kerr response has to be independent of the polarization orientation in an isotropic medium. The factor of two in the cross phase modulation is a consequence of the instantaneous response.

It is clear that linear polarization and circular polarization acquire a simple phase shift due to the Kerr–polarization. Elliptic polarization is rotated. This rotation is exploited in the construction of an artificial saturable absorber with the use of an analyzer.

In lasers with free-space propagation within the Fabry–Pérot resonator, the Kerr–effect can be used to produce intensity dependent focussing. This way of producing an artificial fast saturable absorber is called KLM. As mentioned earlier, KLM was first accomplished by W. Sibbett and his group [22]. The Kerr–effect imposes a spatial intensity dependent phase profile upon the beam propagating through the gain crystal (see Fig. 7). This phase profile leads to intensity dependent beam diameter variations in a Fabry–Pérot resonator with spherical mirror(s) that has been designed to operate close to its instability regime. If the beam diameter is decreased at high intensities within a spatially varying gain profile, as the one produced from a Gaussian pump-beam, the high intensities experience higher gain. This enhancement of gain with increasing intensity is equivalent to saturable absorber action. The proper design of the resonator for optimization of KLM has received a great deal of attention [57].

VI. Mode-Locking in Presence of Group Velocity Dispersion and Kerr–Effect

As shorter and shorter pulses were generated by dye-lasers, it was realized that the group velocity dispersion (GVD) and the self-phase-modulation (SPM) caused by the Kerr–effect of the laser elements could not be ignored. Group velocity dispersion was controlled by the insertion of prism pairs into the laser resonator [58]. Since these new effects become particularly important with ultrashort pulse generation in solid state laser, using APM and KLM, and since these operating schemes are equivalent to a fast saturable absorber, we discuss here only the modified master equation for fast saturable absorber modelocking

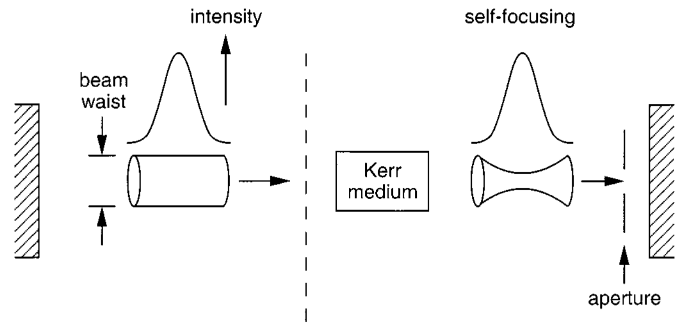

Fig. 7. Artificial saturable absorber realized with Kerr–lens mode-locking.

$$\begin{aligned} \frac{1}{T_R}\frac{\partial}{\partial T} a = &(g-\ell)a + \left(\frac{1}{\Omega_f^2} + jD\right)\frac{\partial^2}{\partial t^2} a \\ &+ (\gamma - j\delta)|a|^2 a. \end{aligned} \tag{30}$$

Here, D is the group velocity dispersion parameter and the filtering action is represented by $(1/\Omega_f^2)(\partial^2/\partial t^2)a$. In a medium of length L, with a propagation constant whose second derivative is β'' the parameter D is $D = \beta'' L/2$. The Kerr–coefficient is $\delta = (2\pi/\lambda)n_2 L/A_{\text{eff}}$, where λ is the carrier wavelength, n_2 is the nonlinear index in cm^2/W and A_{eff} is the effective mode cross-sectional area in cm^2. The gain is taken as time independent as applicable for a gain medium with a long relaxation time. The bandwidth is assumed to be limited by a filter of bandwidth Ω_f. This equation has a simple steady state solution [59]

$$a(t) = A_o \,\text{sech}^{(1+j\beta)}\left(\frac{t}{\tau}\right). \tag{31}$$

The presence of group velocity dispersion raises the possibility of unequal phase and group velocities. When this happens, the carrier phase may slip with respect to the envelope in one roundtrip time. This means that $(1/T_R)\partial a/\partial T = j\psi a$. Introducing this ansatz into (31) and balancing terms one obtains two complex equations.

$$j\psi = g - \ell + \left(\frac{1}{\Omega_f^2 \tau^2} + \frac{jD}{\tau^2}\right)(1+j\beta)^2 \tag{32}$$

$$(2 + 3j\beta - \beta^2)\left(\frac{1}{\Omega_f^2 \tau} + \frac{jD}{\tau}\right) - \frac{1}{2}(\gamma - j\delta)W = 0 \tag{33}$$

where W is the energy in the pulse, $W = 2A_o^2\tau$. We investigate how the pulse parameters vary as one adjusts the group velocity dispersion by a pair of prisms, or the SAM coefficient by changes in the APM or KLM set-up. In such a case the energy W is fixed by the pump level. By taking the real and imaginary parts of (32) and (33) one obtains four real equations for the four parameters: 1) net gain; 2) the phase shift; 3) the pulsewidth; and 4) the chirp parameter.

The solutions of these equations give an overview of all important effects of system parameters on mode-locking. We in-

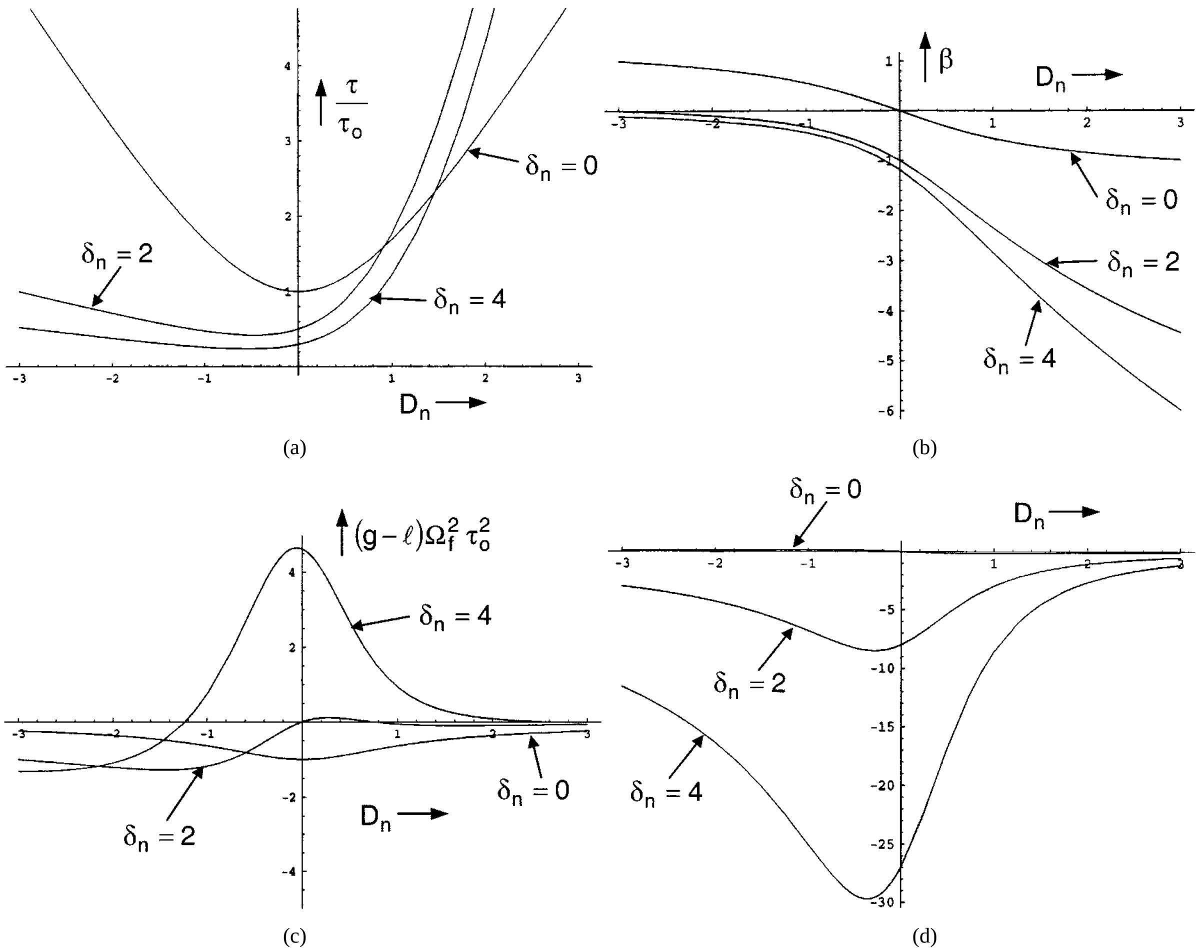

Fig. 8. (a) Pulsewidth. (b) Chirp parameter. (c) Net gain. (d) Phase shift per pass.

vestigate the thought experiment in which the dispersion in the resonator is varied by a prism pair [55], with all other parameters fixed, for several values of the SPM parameter δ. For all solid state lasers, the relaxation time of the gain is much longer than the pulse duration. For fixed pumping, the energy of the pulses remains constant. We use the normalized parameters

$$D_n = D\Omega_f^2, \quad \gamma_n = W\Omega_f^2\tau_o\gamma/6, \quad \delta_n = W\Omega_f^2\tau_o\delta/6 \tag{34}$$

where τ_o is a normalizing pulsewidth. This pulsewidth is chosen conveniently to be the pulsewidth for $\delta = D = 0$, the simple fast saturable absorber mode-locking model. With this choice, $\gamma_n = 2/3$. Separating (32) and (33) into real and imaginary parts, one obtains the set of four equations

$$(g-\ell)\Omega_f^2\tau_o^2 = (\beta^2 - 1 + 2D_n\beta)\left(\tau_o^2/\tau^2\right) \tag{35}$$

$$\psi\Omega_f^2\tau_o^2 = [-D_n(\beta^2-1)+2\beta]\left(\tau_o^2/\tau^2\right) \tag{36}$$

$$-(2-\beta^2) + 3D_n\beta + 3\gamma_n(\tau/\tau_o) = 0 \tag{37}$$

$$D_n(2-\beta^2) + 3\beta + 3\delta_n(\tau/\tau_o) = 0. \tag{38}$$

From (37) and (38) one obtains a relation between the chirp parameter and the pulsewidth:

$$\beta = -\frac{D_n\gamma_n + \delta_n}{1 + D_n^2}\frac{\tau}{\tau_o}. \tag{39}$$

Substituting this expression into (37), we obtain a quadratic equation for the pulsewidth with the (positive) solution:

$$\frac{\tau}{\tau_o} = -(3/2)(s^2\gamma_n - sD_n) + \sqrt{[(3/2)(s^2\gamma_n - sD_n)]^2 + 2s^2} \tag{40}$$

with

$$s \equiv \frac{1 + D_n^2}{D_n\gamma_n + \delta_n}. \tag{41}$$

The pulsewidth as a function of dispersion is plotted in Fig. 8(a). For nonzero SPM, the shortest pulses are obtained with negative dispersion. The pulses are always longer with positive dispersion. The chirp parameter is plotted in Fig. 8(b). A combination of negative dispersion with finite SPM can find a zero chirp solution. For a small SAM coefficient, weak filtering, and negative values of D one finds that the pulse is chirp-free. In this case the pulse is soliton-like, a solution of the nonlinear Schrödinger equation, an approximation to (30)

$$\frac{1}{T_R}\frac{\partial}{\partial T}a = jD\frac{\partial^2}{\partial t^2}a - j\delta|a|^2 a \tag{42}$$

with the chirp-free hyperbolic secant "soliton" solution:

$$a(T,t) = A_o\,\mathrm{sech}\left(\frac{t}{\tau}\right)e^{-j\delta|A_o|^2T/2T_R}. \tag{43}$$

The pulse is continuously phase shifted by the Kerr–effect. The amplitude and pulsewidth obey the "area theorem"

$$|A_o|\tau = \sqrt{2|D|/\delta}. \tag{44}$$

A pulse forms via the balance of GVD and SPM. This behavior is characteristic of fiber ring lasers with uniform dispersion $(D < 0)$. The pulse in the laser may be described as a soliton weakly perturbed by SAM and filtering. The pulse shaping may be attributed to the soliton-process. The SAM action is required solely for stabilization of the pulse against noise buildup in the intervals between the pulses. The filtering selects the pulsewidth by causing a monotonic increase in loss with decreasing pulsewidth. Fig. 8(c) plots the parameter $g - \ell$ which has to be negative if the pulse train is to be stable against buildup of noise between pulses. We see that excessive SPM can lead to instability near zero dispersion and for positive dispersion.

Fig. 8(d) plots the phase shift ψ per pass. If a phase is added upon each pass, the phase increases linearly in time, which corresponds to a frequency shift. Fluctuations of ψ that may be induced by noise cause the spectral lines of the modes to acquire a finite line width. Further, the phase shift per pass causes a change in the phase velocity. The phase velocity has dropped out of the analysis when we resorted to the slowly varying envelope approximation suppressing the carrier. The carrier $\exp(j\omega_o t \pm j\beta_o z)$ does not appear explicitly. This carrier is multiplied by the envelope function. The carrier slips underneath the envelope if

$$\frac{\omega_o}{v_p} L = \beta_o L \neq \frac{\omega_o}{v_g} L + \psi + 2n\pi \tag{45}$$

where n is an integer. Normally one does not care when such a slip occurs, unless the pulse is only a few cycles long. Then, such a slip leads to a change of pulse shape from pulse to pulse and other undesirable effects to be explained later. It should be noted that the inverse group velocity is affected by the Kerr–effect and has an energy dependent contribution.

There is another mechanism that is very important in limiting the pulsewidth shortening and these are the parasitic sidebands first described and explained by Kelly [60]. The soliton is periodically perturbed by the gain, loss, filtering and SAM action. In the process it radiates (generates continuum). If the continuum generated by the soliton is phase matched from pulse to pulse, its energy can build up and drain the soliton. Fig. 9 shows a phase matching diagram in the frequency domain. A spectral component of the continuum at a frequency deviation Ω_s from the carrier frequency is phase delayed in one pass through the resonator by $D\Omega_s^2$, phase advanced if the dispersion is anomalous $(D < 0)$. The soliton pulse and spectrum experience the phase delay $\delta|A_o|^2/2$. The excitation of the continuum is matched if the sum of delay and advance is equal to a multiple of 2π. When this happens, sidebands appear in the pulse spectrum. One such experimental trace is shown in Fig. 10 [30].

The pulse can also be made chirp-free by a proper balance between the SAM coefficient and the filtering action. The pulsewidth increases rapidly with increasing positive values of D. However, the pulse with the broadest bandwidth is found for slightly positive D. When compressed by grating pairs external to the laser, the shortest pulses are generated.

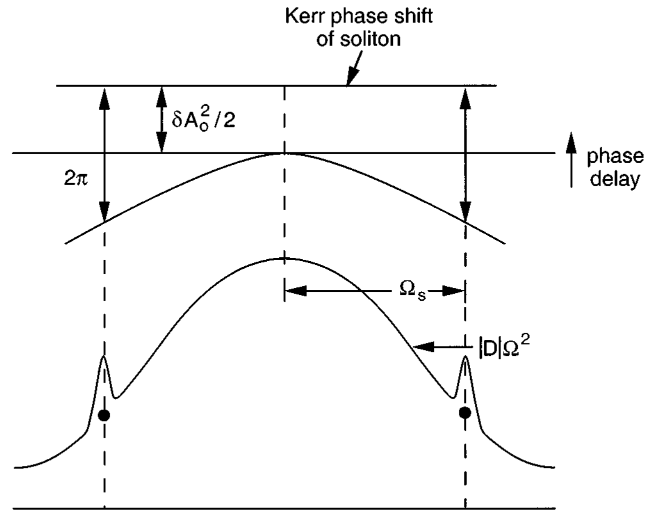

Fig. 9. Phase matching of parasitic sidebands.

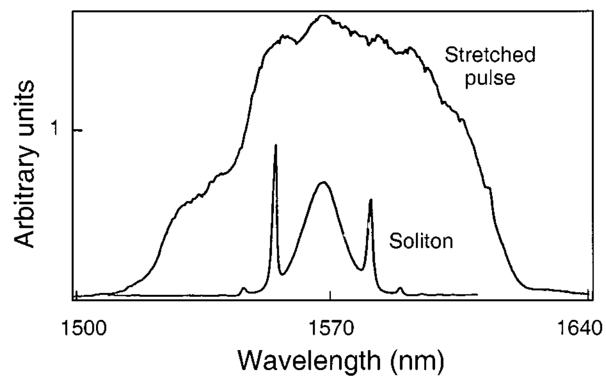

Fig. 10. Spectra of soliton laser and of stretched pulse laser.

VII. The Stretched Pulse-Fiber Laser

Kohichi Tamura, while a graduate student at MIT, was working on fiber ring lasers. When he introduced a novel erbium doped fiber into the ring laser he observed behavior radically different from his first realization of the APM mode-locked fiber ring laser. It turned out that the new erbium doped fiber had positive dispersion. The dispersion around the ring was almost balanced between the positively dispersive erbium fiber and negatively dispersive passive fiber. The pulse circulating in the ring was stretching and compressing by as much as a factor of 20 in one roundtrip. One consequence of this behavior was a dramatic decrease of the nonlinearity and thus increased stability against the Kerr–nonlinearity induced instabilities. No Kelly–sidebands were observed (see Fig. 10). The energy of the output pulses could be increased 100 fold. The minimum pulsewidth was 63 fs, with a bandwidth broader than the erbium gain bandwidth [61]. Fig. 10 shows the spectrum of stretched pulse laser.

A master equation was developed for the operation of the stretched pulse laser [62]. Note had to be taken of the fact that the Kerr–phase shift is produced by a pulse of varying amplitude and width as it circulates around the ring. The Kerr–phase shift for a pulse of constant width, $\delta|a|^2$ had to be replaced by a phase profile that mimics the average shape of the pulse, weighted by its intensity. The SPM action of (6.1) was replaced.

$$\delta|a|^2 \to \delta_o|A_o|^2\left(1-\mu\frac{t^2}{\tau^2}\right). \tag{46}$$

A_o is the pulse amplitude at the position of minimum width. The Kerr–phase profile is expanded to second order in t. The coefficients δ_o and μ are evaluated variationally. The SAM action is similarly expanded. Finally, the dispersion parameter is replaced by the effective dispersion around the ring, D_{net}. The master equation becomes

$$\frac{1}{T_R}\frac{\partial}{\partial T}a = (g-\ell)a + \left(\frac{1}{\Omega_f^2}+jD_{\mathrm{net}}\right)\frac{\partial^2}{\partial t^2}a + (\gamma_o - j\delta_o)|A_o|^2\left(1-\mu\frac{t^2}{\tau^2}\right)a. \tag{47}$$

This equation has Gaussian–pulse solutions. Since the approximations made in arriving at (47) are not applicable to the wings of the pulse, the wings are not Gaussian in fact. A Gaussian–pulse profile in the center of the pulse Fourier–transforms into Gaussian–spectral wings, which decay faster than the wings of a hyperbolic secant square pulse. This helps to suppress the parasitic sidebands.

It should be mentioned that the master equation (46) is a patchwork, it is not an ordinary partial differential equation. The coefficients in the equation depend on the pulse solution. They may be estimated before a solution is found and in a process of successive improved estimates of the coefficients an accurate solution is obtained. The virtue of the equation is that it predicts rather well the general shape of the mode-locked pulse.

VIII. Ultrashort Pulse Generation

We have mentioned that in a fiber ring soliton laser the main pulse shaping mechanism is due to the balance of GVD and SPM. For this to occur, the dispersion must be anomalous. The master equation of the stretched pulse laser (47) also permits steady state solutions without filtering and SAM, as long as $D_{\mathrm{net}} < 0$. These are so called dispersion managed solitons. One may consider this case to be soliton-like, except that the pulse changes its shape periodically in each segment of the resonator of opposite dispersion. Again, one may look at this operation as a perturbed dispersion managed soliton, the SAM and the filtering being small perturbations that stabilize against noise buildup and set the pulsewidth, respectively.

There is one more interesting property of the stretched pulse operation. Dispersion managed solitons may form even when the net dispersion D_{net} as seen by a linearly propagating pulse is zero or slightly positive. This is a surprising result which was discovered in the study of dispersion managed soliton propagation [31]. It turns out that the stretched pulse changes its spectrum in its propagation through the two segments of fiber of opposite dispersion. The spectrum in the segment with normal (positive) dispersion is narrower than in the segment of anomalous (negative) dispersion. The pulse sees an effective net negative dispersion, provided that the positive D_{net} is not too large. In (46), D_{net} is to be replaced by D_{eff} which can be computed

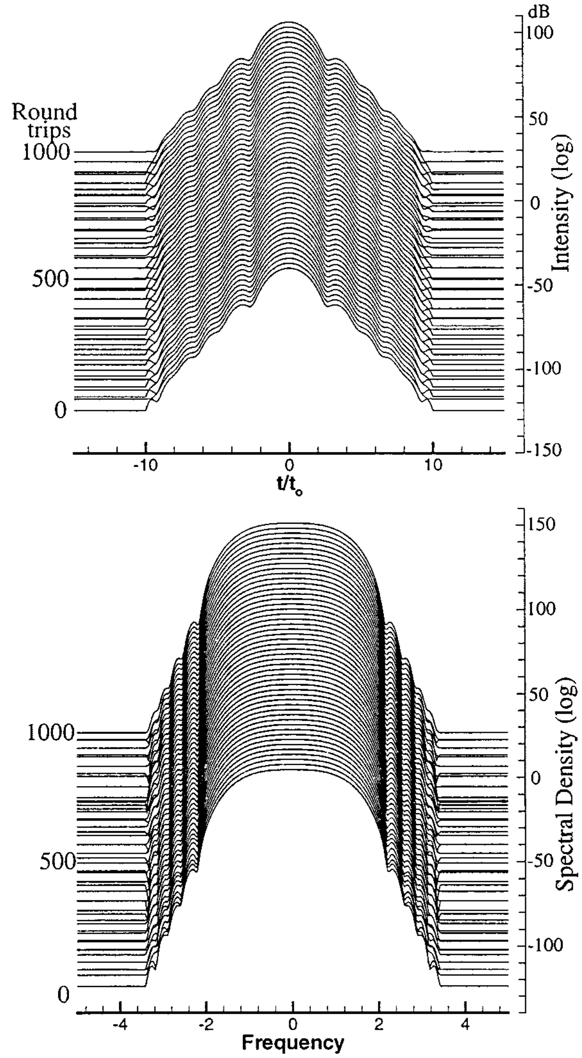

Fig. 11. Intensity profile and the spectrum of dispersion managed soliton at zero net dispersion evaluated at the center of negatively dispersive segments.

variationally. Thus, dispersion managed soliton-like solutions can exist even when D_{net} is zero. However, they exist only if the stretching factor is of the order of two or higher.

When the dispersion managed soliton equation (with no SAM and no filtering) is numerically integrated, solutions are found that resemble the Gaussian–pulses down to about -10 dB from the peak, but then show rather complicated structure (see Fig. 11). The remarkable property of these solutions is that they do not radiate (generate continuum) even though they propagate in a medium with abrupt dispersion changes. This prompted the author to ask the question whether one could view dispersion managed solitons as nonlinear Bloch–waves in a periodic structure, namely waves that reproduce themselves from period to period, and only acquire a net phase shift in the process. It was shown that the Kerr–effect produces a self-consistent scattering potential that makes this kind of solution possible [63].

This brings us to the discussion of the generation of ultrashort pulses. The world record at the moment is that of a Ti : Sapphire laser generating pulses that contain only two cycles [47], [49], [64]. The system is a standard KLM mode-locked system with

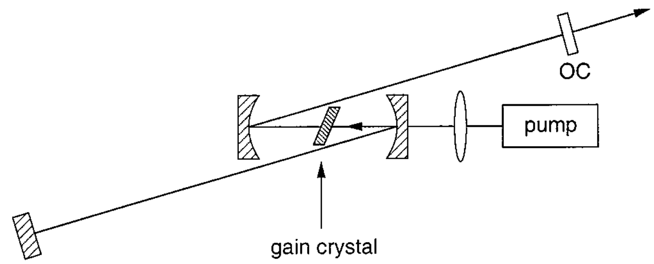

Fig. 12. Laser resonator with dispersion compensating mirrors. The shaded mirrors are double-chriped Bragg–reflectors.

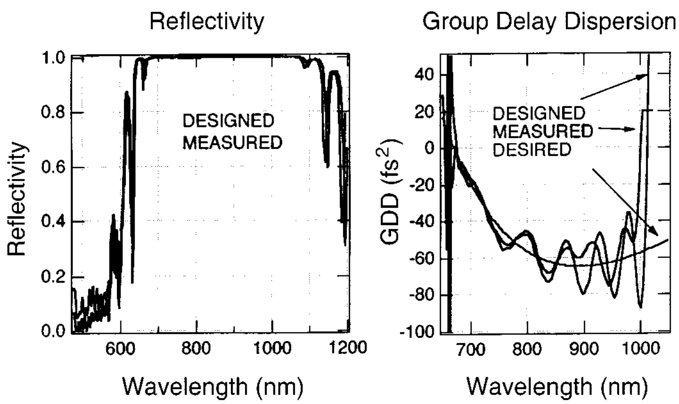

Fig. 13. Reflectivity and group delay of doubly chirped mirror [49].

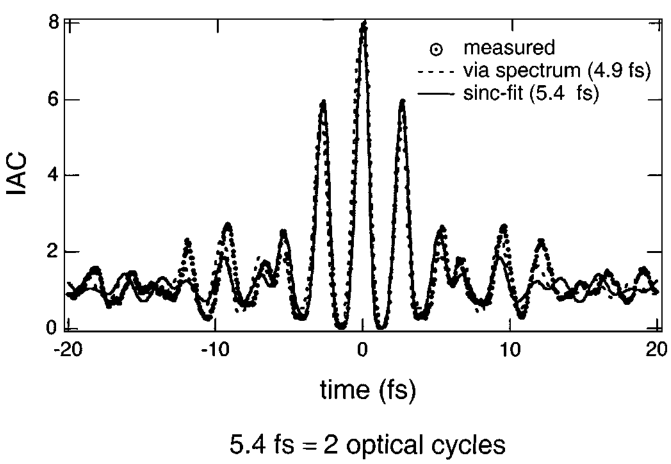

Fig. 14. Autocorrelation of two-cycle pulse.

dispersion compensating mirrors and a prism pair for compensation of higher order dispersion (see Fig. 12). The mirrors are carefully designed to be broad-band and to eliminate as much as possible the fringes that are produced by the Fabry–Pérot resonance between the air-mirror interface and the Bragg–reflector. This is accomplished by a double chirped mirror design [44]. The calculated and measured reflectivity and the group dispersion delay (GDD) are shown in Fig. 13. Fig. 14 shows the measured (interferometric) autocorrelation of the pulse, the autocorrelation inferred from the spectrum under the assumption of constant phase, and a fit assuming a sinc function pulse amplitude [49]. The assumption of a sinc function is an approximation to a pulse that is band-limited by a mirror reflectivity that is a rectangular function of frequency in a resonator with perfectly balanced GVD. Of course, the situation is not that simple. A better model is the stretched pulse, or dispersion managed, soliton propagation.

IX. Conclusion

The advances in mode-locking in the last three and a half decades have been truly remarkable as shown by the historical graph of Fig. 1. Mode-locked Ti : Sapphire, Nd : YAG, erbium-doped fiber lasers and others have been commercialized. The question then arises whether further advances in mode-locked sources are to be expected. The answer is yes. Wavelength regimes will have to be covered in which 10-fs pulses have not been generated as yet. Cr : Forsterite lasers are an example. It is likely that the pulsewidth in these lasers is limited by Raman–excitations in the forsterite crystal that cause parasitic loss [65]. The quest for ultrashort pulses will continue since it still holds great promise. Pulses of two cycles duration spread very rapidly when propagating in any medium, including air. Yet, this does not detract from their usefulness as broad-band sources of spectrum. The demonstration of cellular resolution achieved in optical coherence tomography using the two-cycle Ti : Sapphire laser is a case in point [66]. In this application it is the width of the spectrum of the ultrashort pulses that is of importance.

The spectrum of a mode-locked pulse train of two-cycle pulses at a 100-MHz repetition rate has of the order of a million spectral lines. Control of the repetition rate leads to a spectrum with a million accurately spaced spectral lines for use in frequency standards [67]. There is one obstacle that needs to be overcome—if the phase velocity and group velocity in the laser resonator are not commensurate, as explained in Section VI, the comb of spectral lines spaced by $2\pi/T_R$ is displaced from the origin. The line positions are not controlled. The accurate positioning of the spectral lines is a task pursued by several laboratories [68]–[77].

Acknowledgment

The author gratefully acknowledges Prof. E. P. Ippen's suggestions following his careful reading of the manuscript.

References

[1] K. Gürs and R. Müller, "Breitband-modulation durch Steuerung der emission eines optischen masers (Auskopple-modulation)," *Phys. Lett.*, vol. 5, pp. 179–181, 1963.
[2] K. Gürs, "Beats and modulation in optical ruby lasers," in *Quantum Electronics III*, P. Grivet and N. Bloembergen, Eds. New York: Columbia Univ. Press, 1964, pp. 1113–1119.
[3] H. Statz and C. L. Tang, "Zeeman effect and nonlinear interactions between oscillating laser modes," in *Quantum Electronics III*, P. Grivet and N. Bloembergen, Eds. New York: Columbia Univ. Press, 1964, pp. 469–498.
[4] M. DiDomenico, "Small-signal analysis of internal (coupling type) modulation of lasers," *J. Appl. Phys.*, vol. 35, pp. 2870–2876, 1964.
[5] L. E. Hargrove, R. L. Fork, and M. A. Pollack, "Locking of He–Ne laser modes induced by synchronous intractivity modulation," *Appl. Phys. Lett.*, vol. 5, pp. 4–6, 1964.
[6] A. Yariv, "Internal modulation in multimode laser oscillators," *J. Appl. Phys.*, vol. 36, pp. 388–391, 1965.

[7] H. W. Mocker and R. J. Collins, "Mode competition and self-locking effects in a Q-switched ruby laser," *Appl. Phys. Lett.*, vol. 7, pp. 270–272, 1965.

[8] E. P. Ippen, C. V. Shank, and A. Dienes, "Passive mode locking of the cw dye laser," *Appl. Phys. Lett.*, vol. 21, pp. 348–350, 1972.

[9] C. V. Shank and E. P. Ippen, "Sub-picosecond kilowatt pulses from a mode-locked cw dye laser," *Appl. Phys. Lett.*, vol. 24, pp. 373–375, 1974.

[10] J. G. Fujimoto, A. M. Weiner, and E. P. Ippen, "Generation and measurement of optical pulses as short as 16 fs," *Appl. Phys. Lett.*, vol. 44, pp. 832–834, 1984.

[11] R. L. Fork, B. I. Greene, and C. V. Shank, "Generation of optical pulses shorter than 0.1 psec by colliding pulse mode-locking," *Appl. Phys. Lett.*, vol. 38, pp. 617–619, 1981.

[12] W. H. Knox, R. L. Fork, M. C. Downer, R. H. Stolen, C. V. Shank, and J. A. Valdmanis, "Optical pulse compression to 8 fs at a 5-kHz repetition rate," *Appl. Phys. Lett.*, vol. 46, pp. 1120–1122, 1985.

[13] R. L. Fork, C. H. B. Cruz, P. C. Becker, and C. V. Shank, "Compression of optical pulses to six femtoseconds by using cubic phase compensation," *Opt. Lett.*, vol. 12, pp. 483–485, 1987.

[14] D. I. Kuizenga and A. E. Siegman, "Modulator frequency detuning effects in the FM mode-locked laser," *IEEE J. Quantum Electron.*, vol. QE-6, pp. 803–808, 1970.

[15] E. G. Arthurs, D. J. Bradley, and A. G. Roddie, "Buildup of picosecond pulse generation in passively mode-locked rhodamine dye lasers," *Appl. Phys. Lett.*, vol. 23, pp. 88–90, 1973.

[16] H. A. Haus, "Theory of mode locking with a fast saturable absorber," *J. Appl. Phys.*, vol. 46, pp. 3049–3058, 1975.

[17] G. H. C. New, "Pulse evolution in mode-locked quasicontinuous lasers," *IEEE J. Quantum Electron.*, vol. QE-10, pp. 115–124, 1974.

[18] H. A. Haus, C. V. Shank, and E. P. Ippen, "Shape of passively mode-locked laser pulses," *Opt. Commun.*, vol. 15, pp. 29–31, 1975.

[19] L. F. Mollenauer and R. H. Stolen, "The soliton laser," *Opt. Lett.*, vol. 9, pp. 13–15, 1984.

[20] J. Mark, L. Y. Liu, K. L. Hall, H. A. Haus, and E. P. Ippen, "Femtosecond pulse generation in a laser with a nonlinear external resonator," *Opt. Lett.*, vol. 14, pp. 48–50, 1989.

[21] E. P. Ippen, H. A. Haus, and L. Y. Liu, "Additive pulse modelocking," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 6, pp. 1736–1745, 1989.

[22] D. E. Spence, P. N. Kean, and W. Sibbett, "60-fsec pulse generation from a self-mode-locked Ti : Sapphire laser," *Opt. Lett.*, vol. 16, pp. 42–44, 1991.

[23] D. K. Negus, L. Spinelli, N. Goldblatt, and G. Feugnet, "Sub-100 femtosecond pulse generation by Kerr lens mode-locking in $Ti:Al_2O_3$," *OSA Proc. Advanced Solid-State Lasers*, vol. 10, pp. 120–124, 1991.

[24] F. Salin, J. Squier, and M. Piche, "Mode locking of $Ti:Al_2O_3$ lasers and self-focusing : A Gaussian approximation," *Opt. Lett.*, vol. 16, pp. 1674–1676, 1991.

[25] T. Brabec, C. Spielmann, P. F. Curley, and F. Krausz, "Kerr lens mode-locking," *Opt. Lett.*, vol. 17, pp. 1292–1294, 1992.

[26] M. Piche and F. Salin, "Self-mode locking of solid-state lasers without apertures," *Opt. Lett.*, vol. 18, pp. 1041–1042, 1993.

[27] F. Salin, J. Squier, G. Mourou, M. Piche, and N. McCarthy, "Mode-locking of $Ti:Al_2O_3$ lasers using self-focusing," *OSA Proc. Advanced Solid-State Lasers*, vol. 10, pp. 125–129, 1991.

[28] M. Piche, "Beam reshaping and self-mode-locking in nonlinear laser resonators," *Opt. Commun.*, vol. 86, pp. 156–160, 1991.

[29] U. Keller, W. H. Knox, and G. W. tHooft, "Ultrafast solid-state mode-locked lasers using resonant nonlinearities," *IEEE J. Quantum Electron.*, vol. 28, pp. 2123–2133, Oct. 1992.

[30] K. Tamura, H. A. Haus, and E. P. Ippen, "Self-starting additive pulse mode-locked erbium fiber ring laser," *Electron. Lett.*, vol. 28, pp. 2226–2227, 1992.

[31] J. H. B. Nijhof, N. J. Doran, W. Forysiak, and F. M. Knox, "Stable soliton-like propagation indispersion-managed system with net anomalous, zero, and normal dispersion," *Electron. Lett.*, vol. 33, pp. 1726–1727, 1997.

[32] K. Tamura, E. P. Ippen, H. A. Haus, and L. E. Nelson, "77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser," *Opt. Lett.*, vol. 18, pp. 1080–1082, 1993.

[33] S. Backus, C. G. Durfee III, M. M. Murnane, and H. C. Kapteyn, "High power ultrafast lasers," *Rev. Sci. Instrum.*, vol. 69, pp. 1207–1223, 1998.

[34] C.-P. Huang, M. T. Asaki, S. Backus, M. M. Murnane, H. C. Kapteyn, and H. Nathel, "17-fs pulses from a self-mode-locked Ti : sappire laser," *Opt. Lett.*, vol. 17, pp. 1289–1291, 1992.

[35] I. P. Christov, M. M. Murnane, H. C. Kapteyn, J. Zhou, and C. P. Huang, "Fourth-order dispersion-limited solitary pulses," *Opt. Lett.*, vol. 19, pp. 1465–1467, 1994.

[36] C. Spielmann, P. F. Curley, T. Brabec, and F. Krausz, "Ultrabroadband femtosecond lasers," *IEEE J. Quantum Electron.*, vol. 30, pp. 1100–1114, Apr. 1994.

[37] R. Szipöcs, K. Ferencz, C. Spielmann, and F. Krausz, "Chirped multilayer coatings for broadband dispersion control in femtosecond lasers," *Opt. Lett.*, vol. 19, pp. 201–203, 1994.

[38] A. Stingl, M. Lenzner, C. Spielmann, F. Krausz, and R. Szipöcs, "Sub-10-fs mirror-dispersion-controlled Ti : Sapphire laser," *Opt. Lett.*, vol. 20, pp. 602–604, 1995.

[39] L. Xu, G. Tempea, C. Spielmann, F. Krausz, A. Stingl, K. Ferencz, and S. Takano, "Continuous-wave mode-locked Ti : Sapphire laser focusable to $5 * 10^{13}$ W/cm^2," *Opt. Lett.*, vol. 23, pp. 789–791, 1998.

[40] D. Huang, M. Ulman, L. H. Acioli, H. A. Haus, and J. G. Fujimoto, "Self-focusing-induced saturable loss for laser mode locking," *Opt. Lett.*, vol. 17, pp. 511–513, 1992.

[41] H. A. Haus, J. G. Fujimoto, and E. P. Ippen, "Analytic theory of additive pulse and Kerr lens mode locking," *IEEE J. Quantum Electron.*, vol. 28, pp. 2086–2096, Oct. 1992.

[42] L. E. Nelson, E. P. Ippen, and H. A. Haus, "Broadly tunable sub-500 fs pulses from an additive-pulse mode-locked thulium-doped fiber ring laser," *Appl. Phys. Lett.*, vol. 67, pp. 19–21, 1995.

[43] J. Goodberlet, J. Jacobson, J. G. Fujimoto, P. A. Schulz, and T. Y. Fan, "Self-starting additive-pulse mode-locked diode-pumped Nd : YAG," *Opt. Lett.*, vol. 15, pp. 504–506, 1990.

[44] F. X. Kärtner, N. Matuschek, T. Schibli, U. Keller, H. A. Haus, C. Heine, R. Morf, V. Scheuer, M. Tilsch, and T. Tschudi, "Design and fabrication of double-chipred mirrors," *Opt. Lett.*, vol. 22, pp. 831–833, 1997.

[45] D. H. Sutter, I. D. Jung, F. X. Kärtner, N. Matuschek, F. Morier-Genoud, V. Scheuer, M. Tilsch, T. Tschudi, and U. Keller, "Self-starting 6.5-fs pulses from a Ti : Sapphire laser using a semiconductor saturable absorber and double-chirped mirrors," *IEEE J. Select. Topics Quantum Electron.*, vol. 4, pp. 169–178, Mar./Apr. 1998.

[46] U. Keller, D. A. B. Miller, G. D. Boyd, T. H. Chiu, J. F. Ferguson, and M. T. Asom, "Solid-state low-loss intracavity saturable absorber for Nd : YLF lasers : An antiresonant semiconductor Fabry–Perot saturable absorber," *Opt. Lett.*, vol. 17, pp. 505–507, 1992.

[47] D. H. Sutter, G. Steinmeyer, L. Gallmann, N. Matuschek, F. Morier-Genoud, U. Keller, V. Scheuer, G. Angelow, and T. Tschudi, "Ultrabroadband pulses in the two-cycle regime by SESAM-assisted Kerr-lens modelocking of an all-solid-state Ti : sappire laser," *OSA Trends in Optics and Photonics (TOPS)*, vol. 26, pp. 358–360, 1999.

[48] N. Matuschek, F. X. Kärtner, and U. Keller, "Analytical design of double-chirped mirrors with custom-tailored dispersion characteristics," *IEEE J. Quantum Electron.*, vol. 35, pp. 129–137, 1999.

[49] U. Morgner, F. X. Kärtner, S. H. Cho, Y. Chen, H. A. Haus, J. G. Fujimoto, E. P. Ippen, V. Scheuer, G. Angelow, and T. Tschudi, "Sub-two-cycle pulses from a Kerr-lens mode-locked Ti : Sapphire laser," *Opt. Lett.*, vol. 24, pp. 411–413, 1999.

[50] D. H. Sutter, G. Steinmeyer, L. Gallmann, N. Matuschek, F. Morier-Genoud, U. Keller, V. Scheuer, G. Angelow, and T. Tschudi, "Semiconductor saturable-absorber mirror-assisted Kerr-lens mode-locked Ti : Sapphire laser producing pulses in the two-cycle regime," *Opt. Lett.*, vol. 24, pp. 631–633, 1999.

[51] Y. Chen, F. X. Kärtner, U. Morgner, S. H. Cho, H. A. Haus, E. P. Ippen, and J. G. Fujimoto, "Dispersion-managed mode locking," *J. Opt. Soc. Amer. B*, vol. 16, pp. 1999–2004, 1999.

[52] P. W. Smith, M. A. Duguay, and E. P. Ippen, "Mode-locking of lasers," in *Progress in Quantum Electronics*, J. H. Sanders and S. Stenholm, Eds: Pergamon Press, 1975, pt. 2, vol. 3.

[53] H. A. Haus, "Theory of mode locking with a slow saturable absorber," *IEEE J. Quantum Electron.*, vol. QE-11, pp. 736–746, Sept. 1975.

[54] P.-T. Ho, L. A. Glasser, E. P. Ippen, and H. A. Haus, "Picosecond pulse generation with a CW GaAlAs laser diode," *Appl. Phys. Lett.*, vol. 33, pp. 241–243, 1978.

[55] E. P. Ippen, D. J. Eilenberger, and R. W. Dixon, "Picosecond pulse generation by passive modelocking of diode lasers," *Appl. Phys. Lett.*, vol. 37, pp. 267–269, 1980.

[56] H. A. Haus, "Nonlinear optics," in *Waveguide Optoelectronics*, J. H. Marsh and R. M. De La Rue, Eds. Norwell, MA: Kluwer, 1992, ch. 11.

[57] J. L. A. Chilla and O. E. Martínez, "Spatial-temporal analysis of the self-mode-locked Ti : Sapphire laser," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 10, pp. 638–643, 1993.

[58] R. L. Fork, O. E. Martinez, and J. P. Gordon, "Negative dispersion using pairs of prisms," *Opt. Lett.*, vol. 9, pp. 150–152, 1984.
[59] O. E. Martinez, R. L. Fork, and J. P. Gordon, "Theory of passively mode-locked laser including self-phase modulation and group-velocity dispersion," *Opt. Lett.*, vol. 9, pp. 156–158, 1984.
[60] S. M. J. Kelly, "Characteristic sideband instability of periodically amplified average soliton," *Electron. Lett.*, vol. 28, pp. 806–807, 1992.
[61] K. Tamura, E. P. Ippen, and H. A. Haus, "Pulse dynamics in stretched-pulse lasers," *Appl. Phys. Lett.*, vol. 67, pp. 158–160, 1995.
[62] H. A. Haus, K. Tamura, L. E. Nelson, and E. P. Ippen, "Stretched-pulse additive pulse modelocking in fiber ring lasers: Theory and experiment," *J. Quantum Electron.*, vol. 31, pp. 1–8, 1995.
[63] H. A. Haus and Y. Chen, "Dispersion managed solitons as nonlinear Bloch waves," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 16, pp. 889–894, 1999.
[64] D. H. Sutter, "New frontiers of ultrashort pulse generation," Ph.D. dissertation, Swiss Federal Inst. Technol., Zurich, Switzerland, 2000.
[65] H. A. Haus, I. Sorokina, and E. Sorokin, "Raman-induced real shift of ultrashort mode-locked laser pulses," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 15, pp. 223–231, 1998.
[66] W. Drexler, U. Morgner, F. X. Kärtner, C. Pitris, S. A. Boppart, X. Li, E. P. Ippen, and J. G. Fujimoto, "In vivo ultrahigh-resolution optical coherence tomography," *Opt. Lett.*, vol. 24, pp. 1221–1223, 1999.
[67] J. Ye, L.-S. Ma, T. Daly, and J. L. Hall, "Highly selective terahertz optical frequency comb generator," *Opt. Lett.*, vol. 22, pp. 301–303, 1997.
[68] L. Xu, C. Spielmann, A. Poppe, T. Brabec, F. Krausz, and T. W. Hansch, "Route to phase control of ultrashort light pulses," *Opt. Lett.*, vol. 21, pp. 2008–2010, 1996.
[69] P. Dietrich, F. Krausz, and P. B. Corkum, "Determining the absolute carrier phase of a few-cycle laser pulse," *Opt. Lett.*, vol. 25, pp. 16–18, 2000.
[70] J. Reichert, R. Holzwarth, T. Udem, and T. W. Hänsch, Measuring the frequency of light with mode-locked lasers, in Opt. Commun.. to be published.
[71] T. W. Hänsch, R. Holzwarth, J. Reichert, and Th. Udem, "Precision spectroscopy with femtosecond light pulses," presented at Quantum Electron. Laser Sci. Conf., San Francisco, CA, May 7–12, 2000. paper QWF1.
[72] R. Holzwarth, J. Reichert, Th. Udem, T. W. Hänsch, J. C. Knight, W. J. Wadsworth, and P. St. J. Russel, "Broadening of femtosecond frequency combs and compact optical to radio frequency conversion," presented at Conf. Lasers Electro-Opt., San Francisco, CA, May 7–12, 2000. paper CTuD7.
[73] S. A. Diddams, D. J. Jones, S. T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, and A. J. Stentz, "A direct rf to optical frequency measurement with a femtosecond laser comb spanning 300 THz," presented at Quantum Electron. Laser Sci. Conf., San Francisco, CA, May 7–12, 2000. paper QWF2.
[74] R. J. Jones and J.-C. Diels, "Frequency and phase stabilization of femtosecond light pulses to a Fabry–Perot reference cavity," presented at Quantum Electron. Laser Sci. Conf., San Francisco, CA, R. J. Jones, Ed., May 7–12, 2000. paper QWF3.
[75] G. Steinmeyer, A. E. Dunlop, D. H. Sutter, U. Keller, J. Stenger, and H. R. Telle, "Carrier envelope offset phase stabilization for absolute optical frequency measurement and extreme nonlinear optics," presented at Conf. Lasers Electro-Opt., San Francisco, CA, May 7–12, 2000. paper CWC6.
[76] A. Poppe *et al.*, "Controlling the absolute phase of few-cycle light pulses," presented at Conf. Lasers Electro-Opt., San Francisco, CA, May 7–12, 2000. paper CPD1.
[77] D. Jones *et al.*, "Optical frequency synthesis with an octave bandwidth directly referenced to the cesium standard and based on a single femtosecond laser," presented at Quantum Electron. Laser Sci. Conf., San Francisco, CA, May 7–12, 2000. paper QPD4.
[78] J. Goodberlet, J. Wang, J. G. Fujimoto, and P. A. Schulz, "Starting dynamics of additive pulse mode locking in the $Ti:Al_2O_3$ laser," *Opt. Lett.*, vol. 15, pp. 1300–1302, 1990.

Herman A. Haus (S'50–A'55–SM'58–F'62–LF'91) was born in Ljubljana, Slovenia, in 1925. He attended the Technische Hochschule, Graz, and the Technische Hochschule, Vienna, Austria. He received the B.Sc. degree from Union College, Schenectady, NY, in 1949, the M.S. degree from Rensselaer Polytechnic Institute, Troy, NY, in 1951, and the Sc.D. degree from the Massachusetts Institute of Technology, Cambridge, MA, in 1954.

He joined the Faculty of Electrical Engineering at MIT in 1954, where he is an Institute Professor. He is engaged in research in electromagnetic theory and lasers. He is the author or co-author of six books and over 330 journal articles.

Dr. Haus is a Fellow of the American Physical Society, of the Optical Society of America, and of the Institute of Electrical and Electronics Engineers, a member of the National Academy of Engineering, the National Academy of Sciences, and the American Academy of Arts and Sciences. He is the recipient of the 1984 Award of the IEEE Quantum Electronics and Applications Society, the 1987 Charles Hard Townes Prize of the Optical Society of America, the 1990 Outstanding Paper Award of the IEEE Transactions on Education, the 1991 IEEE Education Medal of the Institute of Electrical and Electronics Engineers, the 1994 Frederic Ives Medal of the Optical Society of America, the President's 1995 National Science Medal, and the Ludwig Wittgenstein-Preis Award of the Austrian Government 1997. He holds honorary doctor's degrees from Union College, Schenectady, New York, the Technical University of Vienna, Austria, and the University of Ghent, Belgium.