Verification of the Magnetic-Dipole Nature of Our Transition: Standing Wave Experiment

Zachary Buckholtz

February 2, 2017

Abstract

Between April and September of 2016 we did several variations of a standing wave experiment in an attempt to verify the magnetic-dipole nature of our transition. Although unsuccessful, the experiments did provide some interesting results.

1 A Brief Introduction: What were we trying to do?

Back in late 2015 and early 2016, we were working on a Rabi flopping experiment on an optical transition that we suspected was a magnetic-dipole transition with the intention of characterizing the transition's interaction with the magnetic field of our beam. We used an intense green laser at 527 nm to induce the flopping behavior and observed the transmission after various durations of flopping. However, it is well known that Rabi flopping can occur due to interactions with the electric field as well [1, 2, 3, 4, 5]. Thus, there was still the need to conclusively verify that our transition was indeed a magnetic-dipole transition and we weren't seeing any electric effects. The fate of the lab was in the balance.

2 Why did we try to do this?

2.1 For the Left-Handed Experiment

This Rabi flopping experiment was part of the longer term left-handed waves project. The goal of the left-handed waves project is to demonstrate propagation of left-handed electromagnetic waves, as opposed to the usual right-handed waves, in an atomic system at optical frequencies. Natural materials do not have the necessary magnetic and electric responses to support such waves, so, in order to observe left-handed waves, we must be able to manipulate these responses. The material that we used in our experiment was EU:Y_2SO_5 (YSO for short).

2.2 Why do we think this transition is a magnetic-dipole transition?

The first order of business, and that which concerns us here in this summary, was to find an atomic transition with a strong magnetic response with no electric response[1]. After looking through the literature and running various codes (Cowan code, RELIC), we identified the $^7F_0 \rightarrow {}^5D_1$ transition in Eu:YSO (referred to as the transition or our transition from now on) as a potential candidate [6]. More evidence came in 2015 when Novotny et. al. [7] showed that this transition was indeed a magnetic-dipole transition. However, their group used a different host crystal and did not measure the magnetic-dipole moment of the transition.

3 How did we try to do this?

The short answer is several different ways. The long answer is

[1]The reason for this is because a materials response to an electric field is generally much strong than its response to the magnetic field. See [6]

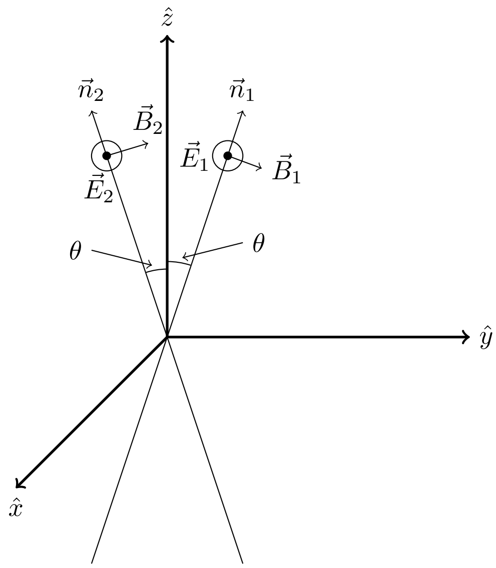


Figure 1: Visualization of the beams. Each beam is described by a propagation direction $\hat{n}$, electric field $\vec{E}$, and a magnetic field $\vec{B}$. The beams are propagating in the yz plane with the z-axis bisecting the angle between propagation directions.

3.1 Standing waves

Our first few attempts at identifying the nature of the transition were centered around an interesting property of standing waves. Imagine two beams of the same frequency that cross each other at an angle 2θ. Without loss of generality, we can put a coordinate system in such that the beams are propagating in the yz plane and each beam's k-vector is an angle θ from the z-axis. See Figure 1.

In the figure, $\hat{n}$, $\vec{E}$, and $\vec{B}$ are the propagation direction, elecric field vector, and magnetic field vector, respectively[2]. We assume that the beams have the same amplitude and the electric fields are both pointing out of the page. The same analysis can be done with the magnetic field pointing out of the page. In this case one would get the same results except the electric and magnetic fields would be switched. In general, there can also be a phase difference between the two beams. We have for the electric fields

$$\vec{E}_1 = E_0 e^{i(k\hat{n}_1\cdot\vec{r}-\omega t)}\hat{x} \tag{1a}$$
$$\vec{E}_2 = E_0 e^{i(k\hat{n}_2\cdot\vec{r}-\omega t+\phi)}\hat{x} \tag{1b}$$

From Figure 1 we see that the propagation directions can be written as

$$\hat{n}_1 = \sin\theta\hat{y} + \cos\theta\hat{z} \tag{2a}$$
$$\hat{n}_2 = -\sin\theta\hat{y} + \cos\theta\hat{z} \tag{2b}$$

When we add the electric fields together we get

$$\begin{aligned}\vec{E}_1 + \vec{E}_2 &= E_0 e^{i(k\cos\theta z-\omega t)}(e^{ik\sin\theta y} + e^{-i(k\sin\theta y-\phi)})\hat{x}\\ &= E_0 e^{i(k\cos\theta z-\omega t)}e^{\phi/2}(e^{ik\sin\theta y-\phi/2} + e^{-i(k\sin\theta y-\phi/2)})\hat{x}\\ &= 2E_0 e^{i(k\cos\theta z-\omega t+\phi/2)}\cos(k\sin\theta y-\phi/2)\hat{x}\end{aligned} \tag{3}$$

We can find the direction of the magnetic field using Faraday's law, $\hat{n}\times\hat{E} = \omega\vec{B}$. Note that we are only considering the direction of the fields here, not the magnitude. The specific relation between the amplitudes of the electric and

[2]We assume right-handed waves throughout.

magnetic fields is not the important part; we only need to know that the electric field amplitude is much larger than the magnetic field amplitude. From this equation, we can see that the directions of the magnetic fields are

$$\hat{B}_1 = \cos\theta\hat{y} - \sin\theta\hat{z} \tag{4}$$

$$\hat{B}_2 = \cos\theta\hat{y} + \sin\theta\hat{z} \tag{5}$$

so the full magnetic fields are

$$\vec{B}_1 = B_0 e^{i(k\hat{n}_1\cdot\vec{r}-\omega t)}(\cos\theta\hat{y} - \sin\theta\hat{z}) \tag{6a}$$

$$\vec{B}_2 = B_0 e^{i(k\hat{n}_2\cdot\vec{r}-\omega t+\phi)}(\cos\theta\hat{y} + \sin\theta\hat{z}) \tag{6b}$$

which when added together give

$$\vec{B}_1 + \vec{B}_2 = 2B_0 e^{i(k\cos\theta z-\omega t+\phi/2)}[\cos\theta\cos(k\sin\theta y - \phi/2)\hat{y} + 2i\sin\theta\sin(k\sin\theta y - \phi/2)\hat{z}]. \tag{7}$$

Taking the real parts of equations 3 and 7 gives us the physical fields,

$$\Re[\vec{E}_1 + \vec{E}_2] = \vec{E} = 2E_0\cos(k\cos\theta z - \omega t + \phi/2)\cos(k\sin\theta y - \phi/2)\hat{x} \tag{8}$$

$$\begin{aligned}\Re[\vec{B}_1 + \vec{B}_2] = \vec{B} =& 2B_0\cos(k\cos\theta z - \omega t + \phi/2)\cos\theta\cos(k\sin\theta y - \phi/2)\hat{y}\\ &+2B_0\sin(k\cos\theta z - \omega t + \phi/2)\sin\theta\sin(k\sin\theta y - \phi/2)\hat{z}.\end{aligned} \tag{9}$$

Equations 8 and 9 can be more easily understood by considering two completely antiparallel beams ($\theta = \pi/2$). Assuming $\phi = 0$ for simplicity, the fields in this case are

$$\vec{E} = 2E_0\cos(\omega t)\cos(ky)\hat{x} \tag{10}$$

$$\vec{B} = -2B_0\sin(\omega t)\sin(ky)\hat{z} \tag{11}$$

This is a very interesting result! It shows that, when the beams are counterpropagating, the spatial maxima of the electric and magnetic fields are $\pi/2$ out-of-phase (so are the temporal maxima, but those will average out in the experiment). If our crystal really did only respond to the magnetic field, we could, in principle, see a standing wave of crystal fluorescence (orange) alternating with the green of our laser due to scattering (our eye and cameras are more sensitive to the electric field).

The challenge with seeing the standing wave pattern of these antiparallel beams is that it is extremely small. In fact, it is by necessity right at the diffraction limit. However, the size of the standing wave pattern can be increased by decreasing θ from $\pi/2$, or, in other words, transition the beams from antiparallel to parallel. As this is done, the counterpropagating component of the k-vector, the y component, begins to decrease corresponding to a larger effective wavelength in the y direction. The caveat is that as θ is decreased the magnetic field gains a component in the y direction. This component is in phase with the electric field and grows as θ is decreased. Therefore we must strike a balance; find a value of θ with an out-of-phase magnetic field standing wave pattern that is large enough to image but isn't washed out by the in-phase component of the magnetic field.

3.2 What Do We Expect to See?

An important question to ask is "just how big is this standing wave going to be?" That's a good question Zach, let me fill you in.

The counterpropagating component of the wave vector for both the electric and magnetic fields is given by $k_y = k\sin\theta = 2\pi n\sin\theta/\lambda$ where n is the index of refraction of the crystal and λ is the free space wavelength of the laser. We must keep in mind, though, that it is the intensity we will be observing. Thus, we must square the field amplitudes which corresponds to doubling the wavevector. This final intensity standing wave pattern can be thought of as having an effective k-vector of $2\pi/\lambda_{standing}$ with a length scale of $\lambda_{standing}$. Equating these wavevectors gives us the relation

$$\frac{2n\sin\theta}{\lambda} = \frac{1}{\lambda_{standing}}. \tag{12}$$

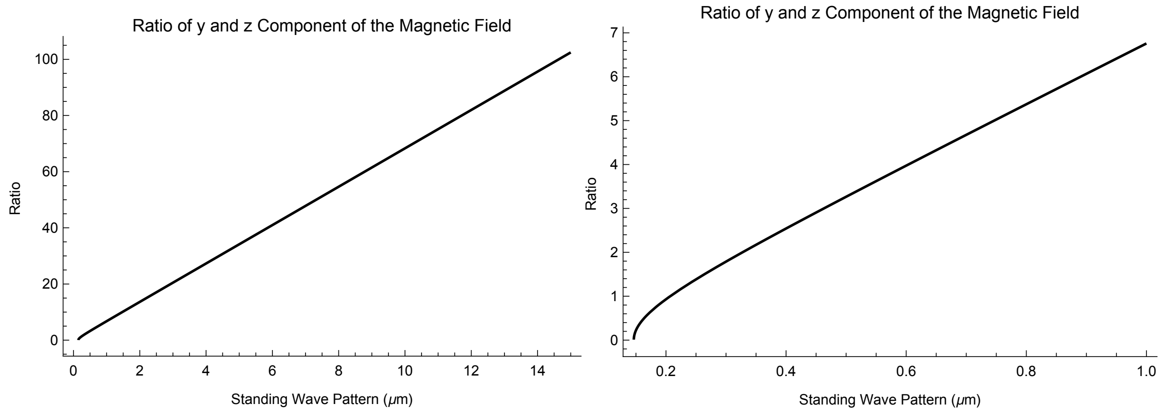


Figure 2: Plot of the ratio of the components of the magnetic field, B_y/B_z. Plot (a) shows the relation becoming linear for large standing wave patterns and (b) is zoomed in to show to show the lower limit behavior.

Since we have an idea of how small of an object we can image, we can set this and solve equation 12 for θ as a function of $\lambda_{standing}$,

$$\theta = \arcsin\left(\frac{\lambda}{2n\lambda_{standing}}\right). \tag{13}$$

The next thing we need to consider is the relative amplitudes of the magnetic field components. What we want is to be able to destinguish fluorescence caused by the magnetic field from any fluorescence that might be caused by the electric field. Because one of the magnetic field components is in phase with the electric field, we must be careful in attributing the source of the fluorescence. Since the beams are oscillating much faster than the response time of our detectors, the terms $\cos^2(k\cos\theta z - \omega t)$ and $\sin^2(k\cos\theta z - \omega t)$ will average out and cancel. Thus the amplitude ratios are

$$\frac{B_y}{B_z} = \cot\theta = \cot\left(\arcsin\left(\frac{\lambda}{2n\lambda_{standing}}\right)\right). \tag{14}$$

This equation can be expanded around small values of $1/\lambda_{standing}$ to show the limiting behavior of the ratio as $\lambda_{standing}$ becomes large.

$$\cot\left(\arcsin\left(\frac{\lambda}{2n\lambda_{standing}}\right)\right) \approx \frac{2n\lambda_{standing}}{\lambda} - \frac{\lambda}{4n\lambda_{standing}} + ... \tag{15}$$

For our values of n and λ this is equal to[3]

$$6.83\lambda_{standing} - \frac{0.073}{\lambda_{standing}} + ... \tag{16}$$

Plots of equation 14 can be seen in Figure 2 depicting this linear behavior. Just for reference, the expansion of θ for large $\lambda_{standing}$ is given by

$$\theta \approx \frac{\lambda}{2n\lambda_{standing}} + ... = \frac{0.146}{\lambda_{standing}} + ... \tag{17}$$

3.3 Standing Wave Experiment Variant 1: Crisscrossed Beams[4]

The idea for this experiment was to try and image the standing wave pattern with the beams almost copropagating (θ small). To see the magnetic field pattern, a long pass filter was used to block the green light and let through the

[3] All lengths are in μm

[4] As it turns out, this was done before I started taking very thorough notes. Since that is the case, most of this was written from memory.

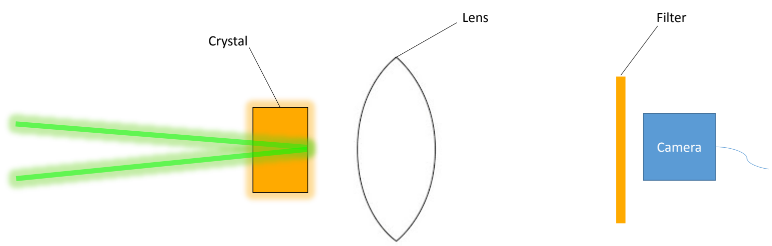


Figure 3: The experimental setup. A different filter was used depending on whether we were trying to observe the electric or magnetic fields. The camera was connected to a computer through a BNC cable. The camera was a Watec ccd camera originally meant for closed circuit security recording.

orange fluorescence. The electric field pattern would be seen using a short pass filter that blocked the orange. In theory, by comparing the recorded intensity patterns of the electric and magnetic field, we would be able to see if the orange pattern lined up with the green pattern like we expect. The imaging was done with a ccd camera.

We used an Air Force resolution target to get an idea of how small of a pattern could be seen using a short focal length lens[5]. Based on the resolution target, the smallest object that could be resolved with the camera/lens combo was about 5 μm. From equation 14, this means that the ratio of the in-phase to the out-of-phase components of the magnetic field was 34.2, which corresponds to an angle of $\theta = 0.029$ or 1.7°. However, since we would actually be measuring intensities, the ratio we would expect to observe with the camera was $34.2^2 = 1170$. The experimental setup can be seen in Figure 3.

Using this setup we weres able to see standing wave patterns. In fact, the electric field standing wave was pretty easy to see; we actually had to use ND filters[6] to cut down on the brightness, see Figure 4a. The magnetic field, however, was harder to see. The in-phase component of the magnetic field, Figure 4b, was barely visible. The factor of 1170 reduction in intensity from the small crossing angle made it impossible to see the out-of-phase component. Better results could have potentially been obtained with a better camera. The Watec camera used in this setup did not allow for ISO control (the ISO automatically adjusted). The camera we used also didn't have any cooling mechanism or use any other noise reduction methods. We did not try this particular setup with the camera from the localization experiment.

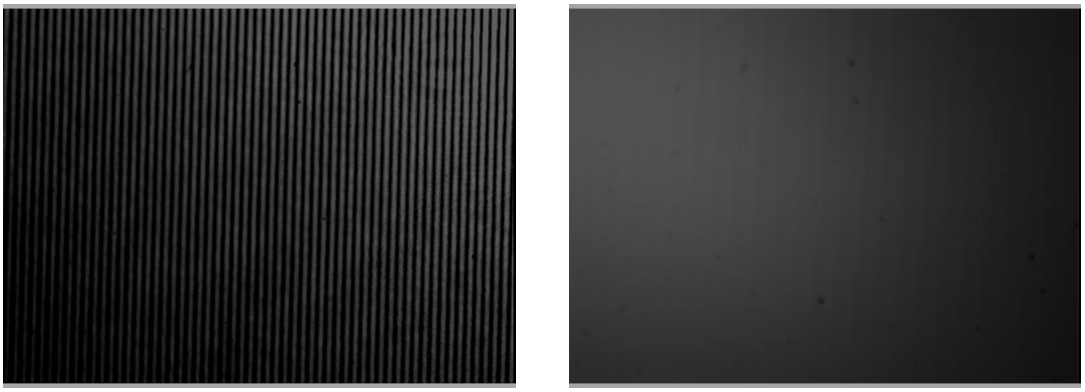
3.4 Standing Wave Experiment Variant 2: A New Hope?

With the setup from the previous section looking like it wasn't going to work, we decided to change things up a bit. Although we couldn't image the standing wave when $\theta = \pi/2$ directly, we still might be able to distinguish the intensity maxima from the minima. The idea was to place a very small aperture between the crystal and the lens in order to select a small slice of the standing wave. Although we might not be able to resolve the aperture, we know what part of the standing wave the light is coming from. The amount of light getting through an aperture of width w is given by

$$\int_x^{x+w} \sin^2(k\theta)d\theta = \frac{w}{2} - \frac{1}{2k}(\sin(x+w)\cos(x+w) - \sin(x)\cos(x)) \tag{18}$$

[5] I don't remember what the focal length was. I do remember that it did not attach to the camera. The one that did attach to the camera wasn't strong enough. Improving the lens setup would go a long way towards improving the results of this experiment if it were to be done again.

[6] I don't rememebr what the strength of the ND filter was though.



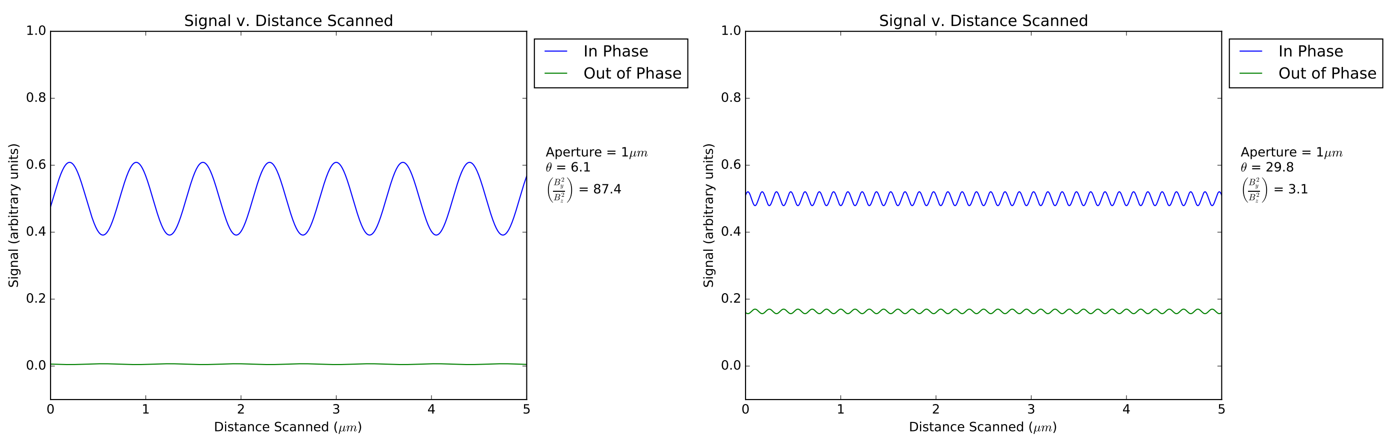
(a) (b)

Figure 4: Images taken with the setup in Figure 3. Figure 4a is the electric field standing wave after going through an ND filter and Figure 4b is the in-phase magnetic field pattern. The out-of-phase pattern could not be imaged this way.

where k is the wavevector of the standing wave pattern and x is the position of the aperture. If this expression gives the in-phase component, the out-of-phase component is given by

$$\frac{w}{2} + \frac{1}{2k}(\sin(x+w)\cos(x+w) - \sin(x)\cos(x)) \tag{19}$$

Both of these expressions are in arbitrary units. See Figure 5 for more details.



(a) (b)

Figure 5: The expected in-phase and out-of-phase signals for different values of θ with the same size aperture. In both plots, the out-of-phase signal is divided by the intensity ratio. Using a small value of θ can lead to a high contrast for the in-phase signal, however due to the large ratio of intensities, the contrast of the out-of-phase signal is much smaller (Figure 5a). At a larger angle, the out-of-phase component has higher contrast at the expense of lower contrast for the in-phase component and a much shorter signal period, which requires finer control of the aperture (Figure 5b)

We went through numerous iterations of this experiment while we smoothed out all the issues that came up. In order to get a decent signal, there must be a balance between having a large enough out-of-phase signal, having fine enough control over the aperture location, and not having an aperture width that is a multiple of the standing wave scale (equations 18 and 19 are zero in this case), see Figure 5. In the last iteration we overcame these difficulties by using a 1 μm slit glued to a piezo and set $\theta = 0$. With $\theta = 0$ the out-of-phase component is maximized and using a slit instead of of a pinhole increased the signal. Putting it on a piezo meant that we could have very fine control over the displacement of the slit. The piezo also, at least in theory, helped us overcome another problem. Since our standing wave was so small, even small mechanical vibrations could throw off our measurements. The piezo would allow us to move the slit faster than these vibratons. The presence of vibrations also required us to measure the magnetic and electric field standing waves at the same time. We did this by using a beam splitter, a couple of filters, and a couple of photon counters, see Figure 6.

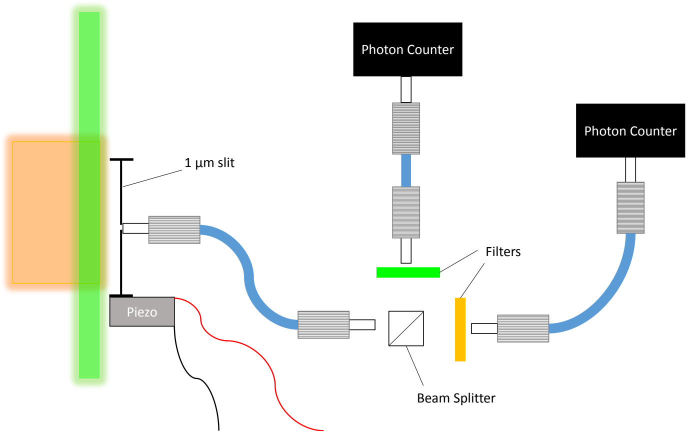


Figure 6: Experimental Setup. The actual setup included some mirrors to align the fibers. The piezo was driven at many different frequencies. As we changed the driving freqency, it became apparent that there was some resonance with the table, which added to the noise in the signal. This setup was on a separate table from the table with the laser and SHG cavity on it. This was to prevent the piezo vibrations from adding more noise by disrupting the incoming laser.

Dispite our best efforts, we were not able to see the magnetic field standing wave. The conclusion was that there were two reasons for this: the signal was too small and there were too many vibrations caused by driving the piezo. However, we were able to see the electric field, although just barely. This was confirmed by blocking the beam and seeing the signal on the oscilloscope change. If we were to try this again, the best bet would be to try a larger value of θ to increase the amplitude of the out-of-phase component as suggested in Figure 5b.

References

[1] D. J. Griffiths. *Introduction to Quantum Mechanics* (Pearson Education, Inc., Upper Saddle River, NJ, 2005).

[2] R. Shankar. *Principles of Quantum Mechanics* (Springer Science+Business Media , Inc., New York, NY, 1994).

[3] J. J. Sakurai, J. Napolitano. *Modern Quantum Mechanics* (Addison-Wesley, San Francisco, CA, 1994).

[4] L. D. Landau, E. M. Lifshitz. *Quantum Mechanics (Non-relativistic Theory)* (Butterworth-Heinemann, Burlington, MA, 2003).

[5] M. O. Scully, M. S. Zubairy. *Quantum Optics* (Cambridge University Press, Cambridge UK).

[6] Z. J. Simmons, *Progress Toward s Negative Refractive Index in an Atomic System: Spectroscopy and Simulations of a Rare-Earth Doped Crystal.* PhD thesis, UW Madison, 2015.

[7] M. Kasperczyk, S. Person, D. Ananias, L. D. Carlos, and L. Novotny, *Excitation of Magnetic-Dipole Transitions at Optical Frequencies*, Phys. Rev. Lett. 114, 163903 (2015).