Physics 625

Fizeau

FIZEAU: Based on interference

Suppose there are two light sources emitting monochromatic light of the same frequency (ν). The sources are separated by a distance >> (λ) = wavelength of light, and the detector is far enough from the sources to treat the sources as plane waves. If $\vec{E}_1$ and $\vec{E}_2$ are the electric fields of the two light sources, the irradiance at detector point (P) is:

$$I = \langle \vec{E}_1^2 \rangle_T + \langle \vec{E}_2^2 \rangle_T + 2 \langle \vec{E}_1 \cdot \vec{E}_2 \rangle_T$$

What is $\langle\ \rangle_T$? $\langle f(t) \rangle_T = \frac{1}{T} \int_t^{t+T} f(t')\,dt'$

For us, the time interval (T) is >> period of light $(= \frac{1}{\nu})$

Physics 625

Fizeau and Schlieren

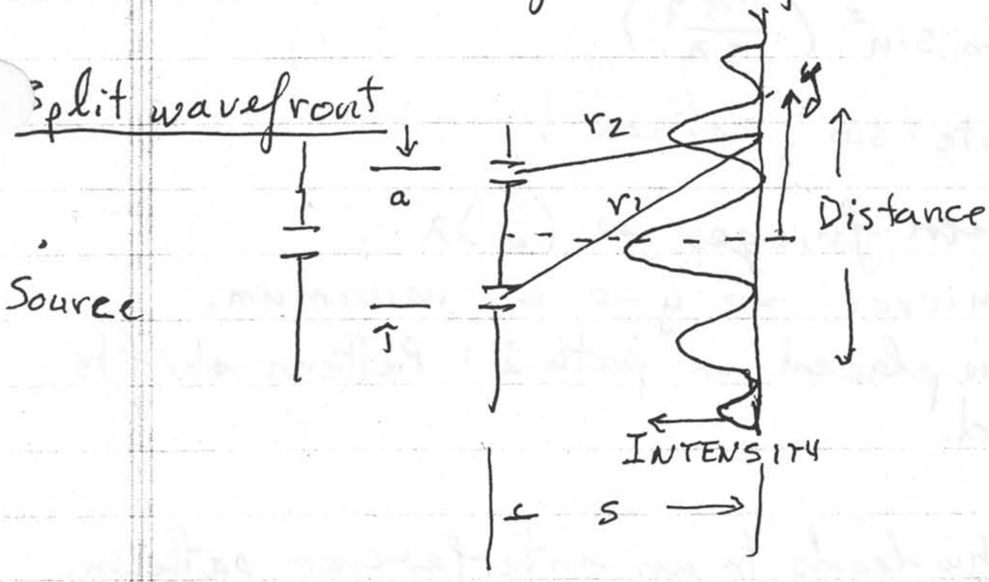
Naturally, if $\vec{E}_1 \perp \vec{E}_2$, there is no interference. When $\vec{E}_1 \parallel \vec{E}_2$, the interference term:

$$2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle_T = \vec{E}_{01} \cdot \vec{E}_{02} \cos(\delta)$$

$\delta = \vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \varepsilon_1 - \varepsilon_2$ is the phase difference due to both the path length $(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r})$ and initial phase difference $(\varepsilon_1 - \varepsilon_2)$.

Now, with continuous lasers the spatial and temporal coherences are long compared to the response time (T) of the detectors. What about before 1948 - no lasers? How to get coherence needed for interference?

Split wavefront



$$r_1 - r_2 \cong \frac{ya}{s}$$

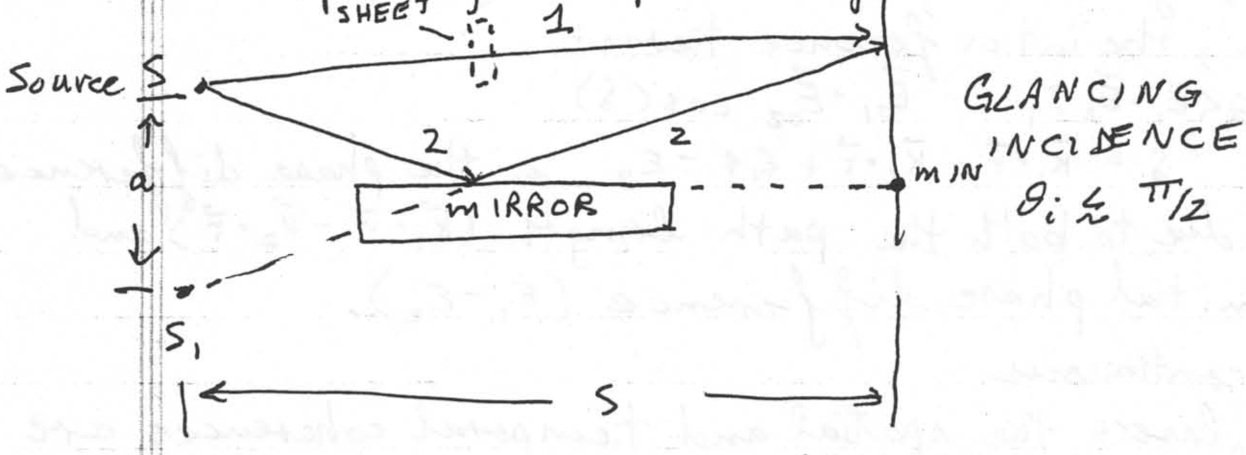
Interference intensity $I = 4 I_0 \cos^2\left(\frac{k(r_1 - r_2)}{2}\right)$

$$= 4 I_0 \cos^2\left(\frac{ya\pi}{s\lambda}\right)$$

Physics 625

Fizeau and Schlieren

Other wave-splitting example: Lloyd's mirror



At glancing incidence: Reflected beam has 180° phase shift

$$\Rightarrow \delta = k(r_1 - r_2) \pm \pi$$

$$\Rightarrow I = 4 I_0 \sin^2\left(\frac{\pi a y}{s \lambda}\right)$$

Note: $\sin^2$, not $\cos^2$!

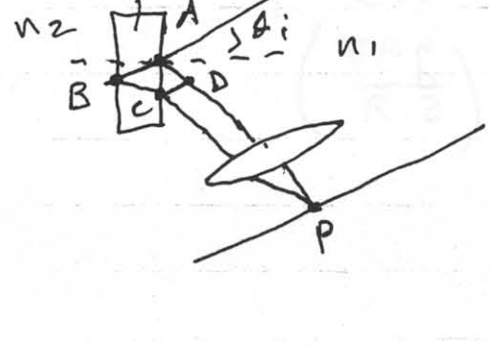
Spacing between fringes is $\left(\frac{s}{a}\right)\lambda$

Parallel to mirror $\Rightarrow y = 0 \Rightarrow$ MINIMUM.

If a sheet is placed in path 1: Pattern shifts upward.

Split Amplitude: This leads to an interference pattern, to bright and dark fringes.

Equal inclination



d = film thickness

Physics 625

Fizeau and Schlieren

Difference in optical path length $= \Lambda = n_f(\overline{AB}) + n_f(\overline{BC}) - n_1(\overline{AD})$

$$\overline{AB} = \overline{BC} = \frac{d}{\cos\theta_t}$$

$$\overline{AD} = \overline{AC}\,\sin\theta_i = \overline{AC}\left(\frac{n_f}{n_1}\right)\sin\theta_t$$

$$\overline{AC} = 2d\tan\theta_t$$

$$\Rightarrow \Lambda = 2n_f\, d\cos\theta_t$$

There is also a phase shift due to the reflections:

$0^\circ \leq \theta_i \lesssim 30^\circ$ two beams have a (π) relative phase shift

So if the phase shift due to the path difference is $(k_0\Lambda)$, the total phase shift is:

$$\delta = k_0\Lambda \pm \pi = \frac{4\pi d}{\lambda_0}\left(n_f^2 - n^2\sin^2\theta_i\right)^{1/2} \pm \pi$$

Interference maximum: when $\delta = 2\pi m$

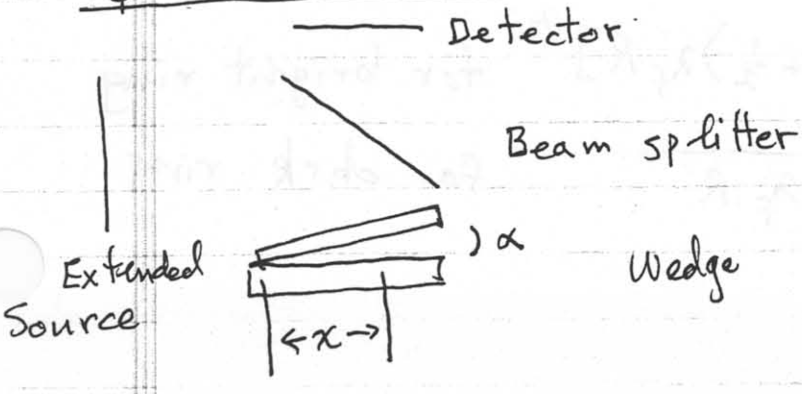
$$\Rightarrow \text{maxima at: } d\cos\theta_t = (2m+1)\left(\frac{\lambda_f}{4}\right)$$

$$m = 0, 1, 2, \ldots$$

$$\lambda_f = \frac{\lambda_0}{n_f}$$

Here (θ_i) is dominant effect. In other situations, $(n_f\, d)$ is dominant.

Equal thickness



View near normal incidence

$\Rightarrow$ Fizeau fringes

Physics 625

Fizeau and Schlieren

Fizeau fringes (cont.)

$$\Lambda = 2 n_f d \cos(\theta_t)$$

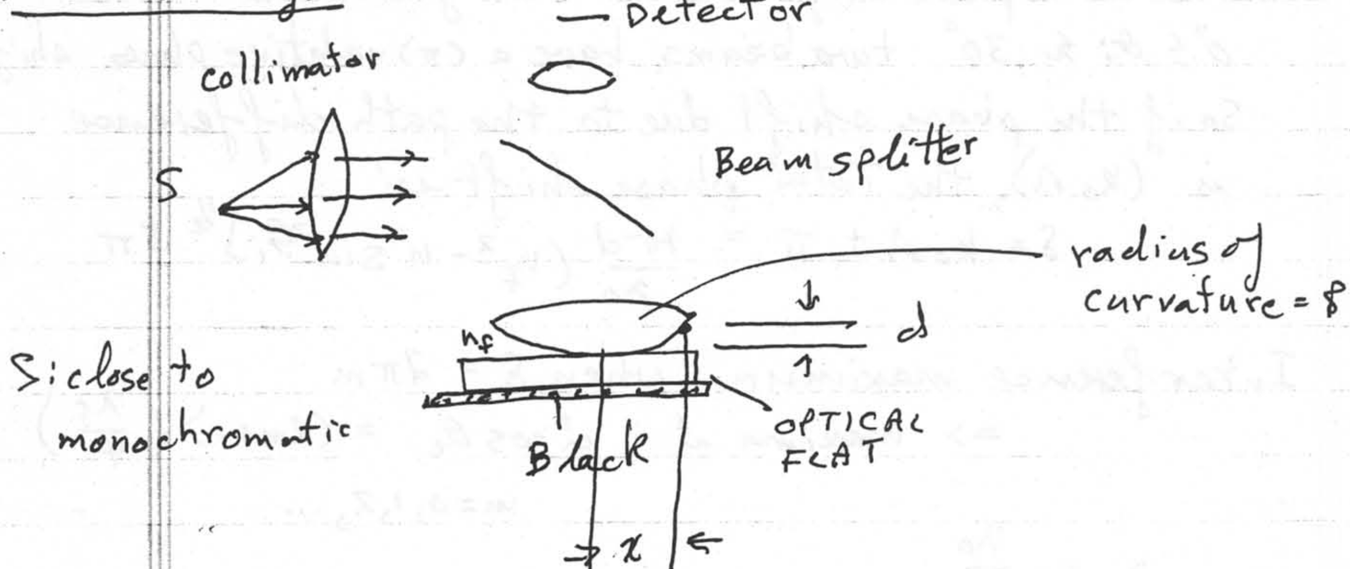
$$d = x\alpha$$

$\Rightarrow$ maxima at: $(m+\frac{1}{2})\lambda_0 = 2 n_f d_m$

or $(m+\frac{1}{2})\lambda_0 = 2\alpha x_m n_f$

$$\Rightarrow x_m = \left(\frac{m+\frac{1}{2}}{2\alpha}\right)\lambda_f$$

Newton's rings



$$x^2 = R^2 - (R-d)^2 = 2Rd - d^2$$

$R >> d$

$$x^2 \cong 2Rd$$

Maxima at: $2 n_f d_m = (m+\frac{1}{2})\lambda_0$

$\Rightarrow x_m = \left[(m+\frac{1}{2})\lambda_f R\right]^{1/2}$ for bright ring

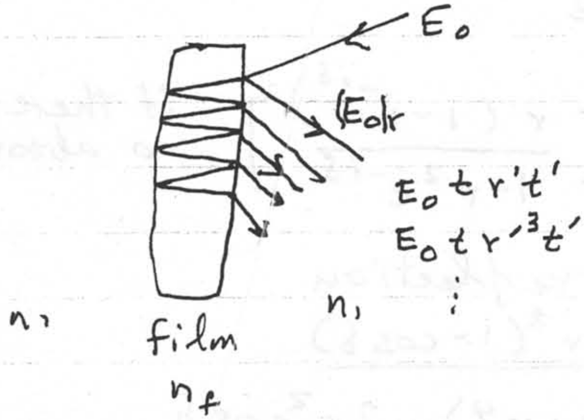
$= \sqrt{m \lambda_f R}$ for dark ring

Physics 625

Fizeau and Schlieren

<u>Wedge - sort of - again</u>: Multiple beams -

What if we have more than one beam? Example:



$r = -r'$ (phase shift)

Between adjacent rays : Λ = optical path length difference
$= 2 n_f d \cos\theta_t$

If $\Lambda = m\lambda$, 2nd, 3rd, 4th, ... rays are in phase, but the 1st ray is (π) out of phase.

$$\Rightarrow (E_{TOTAL})_r = (E_0) r - (E_0 t r t' + E_0 t r^3 t' + E_0 t r^5 t' + \dots)$$
$$= E_0 \{ r - t r t' [1 + r^2 + r^4 + \dots] \}$$
$$= E_0 \left\{ r - \frac{t r t'}{1 - r^2} \right\}$$

$$t t' = 1 - r^2$$

$\Rightarrow (E_{TOTAL})_r = 0$ when $\Lambda = m\lambda$.

When $\Lambda = (m + \frac{1}{2})\lambda$:

1st and 2nd rays are in phase

all others adjacent rays are ($\frac{\lambda}{2}$) out of phase

$$\Rightarrow (E_{TOTAL})_r = E_0 r + E_0 r t t' \{ 1 - r^2 + r^4 - \dots \}$$
$$= E_0 r \left[1 + \frac{t t'}{1 + r^2} \right] = \frac{2 r E_0}{1 + r^2}$$

Physics 625

Fizeau + Schlieren

Consider a more general multiple beam interference

$$E_{1r} = E_o r e^{i\omega t}$$

$$E_{2r} = E_o r t t' e^{i(\omega t - \delta)}$$

$$E_{3r} = E_o r^3 t t' e^{i(\omega t - 2\delta)}$$

$$\vdots$$

$$\Rightarrow (E_{TOTAL})_r = E_o e^{i\omega t}\left[\frac{r(1-e^{-i\delta})}{1-r^2 e^{-i\delta}}\right]$$ if there is no absorption

$\Rightarrow$ flux density in reflection

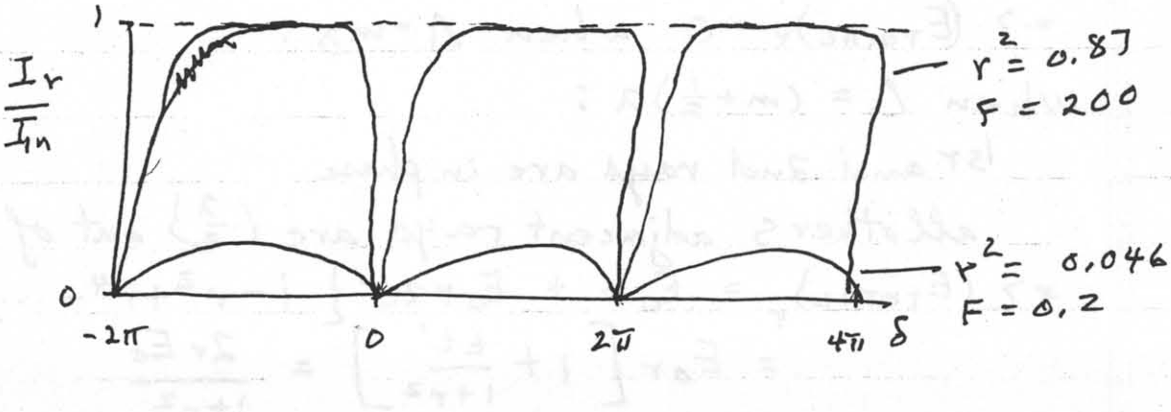
$$I_r = I_{IN} \frac{2r^2(1-\cos\delta)}{(1+r^4)-2r^2\cos\delta}$$

The minimum for I_r occurs when $\delta = 2\pi m$
and maximum for I_r occurs when $\delta = (2m+1)\pi$

Another term used is the <u>coefficient of finesse</u>

$$F = \left(\frac{2r}{1-r^2}\right)^2$$

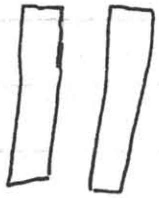
then: $$\frac{I_r}{I_{IN}} = \frac{F\sin^2(\delta/2)}{1+F\sin^2(\delta/2)}$$



Physics 625

Fizeau and Schlieren

Parallel flats



→| |← s

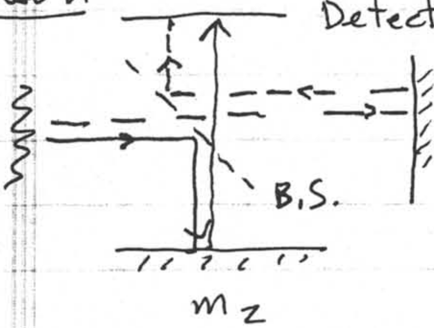
If the flats are parallel to each other and $s >> \lambda$ there is a pattern of bull's-eye fringes -- that moves as the eye moves – as those the fringes are at ∞.

Source is diffuse.

There are several interferometers that have the same general idea:

- Divide the light into 2 beams, typically by using a 45° beamsplitter.
- Include the perturbation in one beam
- Recombine the two beams, typically by using another 45° beamsplitter
- The perturbation shows up as interference fringes.

Michelson



Main difficulty: Initial alignment to make images of M_1 and M_2 exactly parallel and with exactly zero spacing between them.