Physics 625, Etendue and Coupling Flux Handling Devices

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1 Theory of Etendue

The following 2 subsections of theory are from the book by Miller & Roesler (chapter 3/1).

1.1 Etendue, U

An extremely important geometrical concept is known as **etendue**, U, the product of the two previous quantities, area and solid angle, which takes into account both area elements in Figure 3/1-1. Etendue is the projected area **s** of the source times the solid angle Ω of the receiving surface as seen from the source, written $\mathbf{s}(\mathbf{a}/\mathbf{r}^2)$. It is also the projected area **a** of the receiving surface times the solid angle ω of the source as seen from the receiving surface. Etendue has the SI units $m^2.steradian$ and is usually given the unit $(mm^2.sr)$ for maximum clarity, although, since the steradian is dimensionless, (mm^2) is acceptable.

$$\mathbf{U} = \mathbf{s}\Omega = \mathbf{a}\omega = \frac{\mathbf{sa}}{\mathbf{r}^2} \quad (units\ of\ m^2.sr\ or\ m^2\ or\ mm^2\ etc.) \tag{1}$$

Make circles of the thumb and forefinger of each of your hands, space them apart along the line of sight, then wave your eye around to see through both holes while looking along a variety of possible light rays to visualize the concept of etendue. Etendue is a way of quantifying the complicated bundle of all the light rays that can make it through **both** apertures. The design decisions of most optical instruments automatically define a value of etendue, an example being the bundle of rays passing through both the objective lens (aperture) and field stop of a telescope.

1.2 Conservation of etendue

The concept of etendue is particularly interesting and powerful because it is **conserved**; in no way can you increase it by lens magnification or minification:

$$Etendue\ U\ \ is\ \ conserved \tag{2}$$

Neither can you decrease the etendue with lenses – unless, of course, you **obstruct** some of the rays by introducing a third aperture that is too small to accommodate all of the rays, in which case you have really redefined the system stops by this new aperture. There are a number of approaches for showing that the etendue of a bundle of rays is unchanged by lenses, but the simple sketch of Figure 3/1-2 will suffice for our purposes.

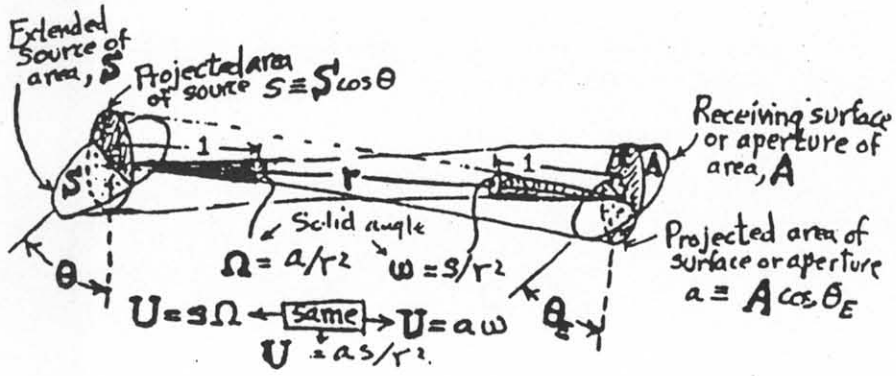


Figure 3/1-1. General key to geometrical concepts of area S,A; projected area s,a; solid angle ω,Ω; and etendue, U.

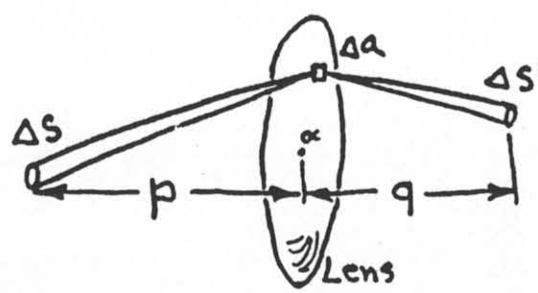


Figure 3/1-2. Concept of conservation of etendue, U. The magnification of elements of object area Δs is neatly compensated by distances p,q, leaving the element of solid angle and hence the element of etendue unchanged by the lens action.

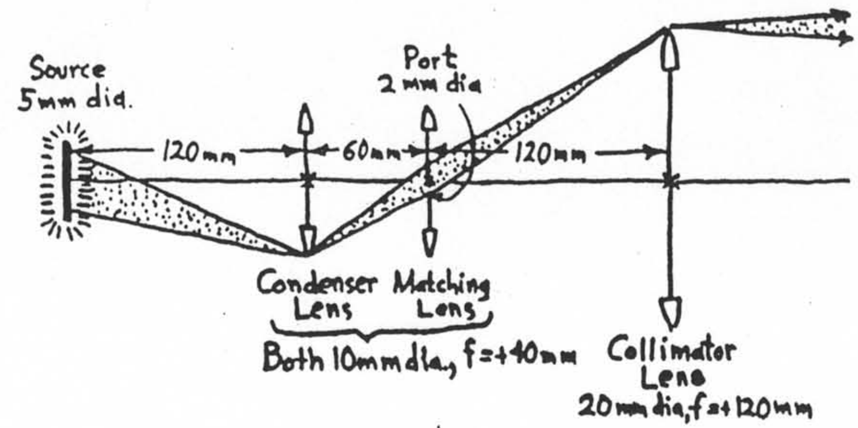


Figure 3/1-3. Solution of etendue example. Ideally one would like to add a matching lens (shown dotted) placed at the 2 mm input hole.

1½

Consider an element of aperture area $\Delta\mathbf{a}$ on the surface of a lens, and an element of object area $\Delta\mathbf{s}$, at distance $\mathbf{p}$ and subtending a solid angle $\Delta\mathbf{s}/\mathbf{p}^2$. The image area Δs' at distance $\mathbf{q}$ will be magnified by $\mathbf{q}/\mathbf{p}$, or in terms of area, Δs' is $\Delta\mathbf{s}(\mathbf{q}/\mathbf{p})^2$, from which the emerging solid angle is $\Delta\mathbf{s}(\mathbf{q}/\mathbf{p})^2/\mathbf{q}^2$ or $\Delta\mathbf{s}/\mathbf{p}^2$, so that the incoming and outgoing elements of solid angle remain the same. Since the bit of lens area $\Delta\mathbf{a}$ is obviously the same bit for incoming and outgoing bundles, the element of etendue ($\Delta\mathbf{a}$ $\Delta\mathbf{s}/\mathbf{p}^2$) also remains the same after passing the lens. Well if each small portion of the ray bundle – each **element** of etendue – is unchanged by passing the lens, so too is the totality **U** of all such ray bundles unchanged by the lens.

Thus, in an optical system, the components (mirror or lens and aperture) which give the lowest etendue act as a bottle neck. In other words, the system cannot transmit more light than allowed by the components which have the smallest etendue.

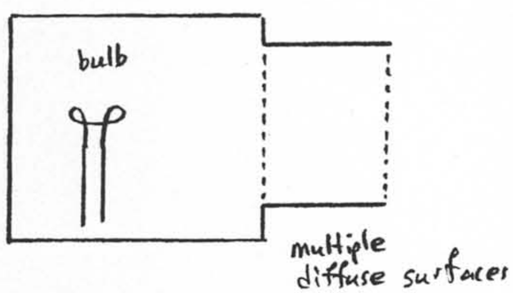
2 Equipment Provided

1. **Extended source.**

   Assume that the Source Luminance is

   $L = 0.010\ bulb/(mm^2.sterad)$ where the "bulb" is an arbitary unit.

   The source transmits the light through 2 separate internal diffuse surfaces to give a uniform Luminance L. WARNING! The lamp housing gets hot.

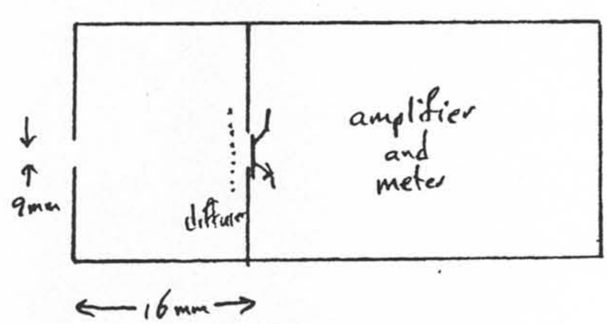


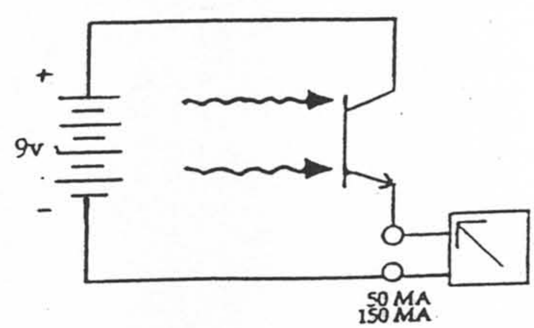
2. **Photo Detector**

   NPN phototransistor with diffusers to give uniform spatial response. The sensitive area of the internal phototransistor base is 4.0 mm diameter. The detector has no external or internal lenses. A 9 mm diameter stop is permanently fixed 16 mm from the sensitive surface.

   This has 3 ranges X1, X5 and X10. After subtracting any zero error, multiply the meter reading by 1, 5 or 10 to get the "signal" S.

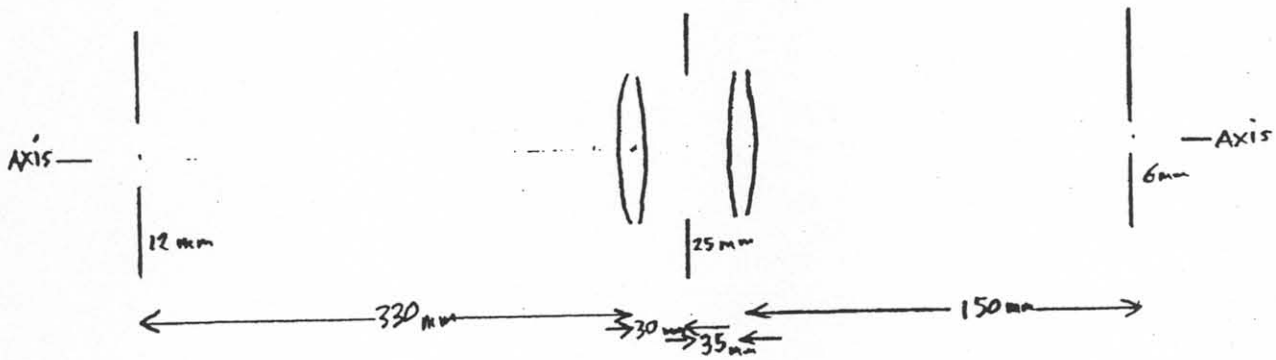
   DO NOT ALTER THE ALIGNMENT OF THE DETECTOR ASSEMBLY!





3. **An "Instrument"**
   This is a set of lenses and apertures, which represent a typical flux processing "Instrument" and allow us to measure the etendue.
   (We sometimes call the "instrument" a NULLOMETER because, by itself, it measures nothing.)
   This typical "instrument" is on a horizontal plate 515 mm x 75 mm and has fixed internal alignment - do not adjust it.
   Light normally

   - enters through the 12 mm ENTRANCE APERTURE on the left,
   - is collimated by a left lens, to be nearly parallel light, then
   - passes through the processing element and then
   - passes through a 25 mm aperture (well call it the PROCESSOR APERTURE PA).
   - Finally, a focusing right lens images the entrance aperture
   - onto the 6 mm EXIT APERTURE on the right.



4. **Apertures**
   3, 6, 9, 12, 18, and 25 mm.

5. **Lenses**
   Focal lengths 25, 40, 65, 90, and 150 mm.

6. **Screen**

7. **Assorted**
   holders, black shroud for aperture, small miror, etc.

This experiment explores some concepts and techniques, as well as difficulties and solutions, which are encountered in working with various flux measuring systems. Not all of the constraints found in this experiment will be relevant to a given situation in the real world, of course. In many cases things may be considerably easier; in other cases, more difficult.

You will probably find that there is more suggested to do than you can comfortably cover in one session. Sections marked with "Optional" can be skipped without impacting on later steps, particularly if you think you understand what is going on. Sections marked (are strictly optional asides, which you might try if they strike your fancy.

3 EXPERIMENTS WITH THE "INSTRUMENT"

1. Inspect the nullometer or "Instrument" carefully, making sure you thoroughly understand its optics. It is essential to make a ray diagram for the instrument in your notebook.

   Left aperture to left lens = 300 mm
   left lens to central aperture = 30 mm
   central aperture to right lens = 35 mm
   right lens to right aperture = 150 mm

   Send light into the instrument alternately through the entrance and exit apertures, and use a small mirror between either lens and the processing stop to show by autocollimation that the apertures are in the focal planes of the lenses between the aperture and the processing stop. Are the entrance and exit aperture diameters in the ratio of the focal lengths of the lenses facing them?

2. Calculate the etendue U of the input and output sections of the instrument using careful measurements of the stop diameters, positions, and lens focal lengths and positions. The instrument has been designed so these are matched. If you have correctly analyzed the geometry your etendue measurements should agree within about ten percent.

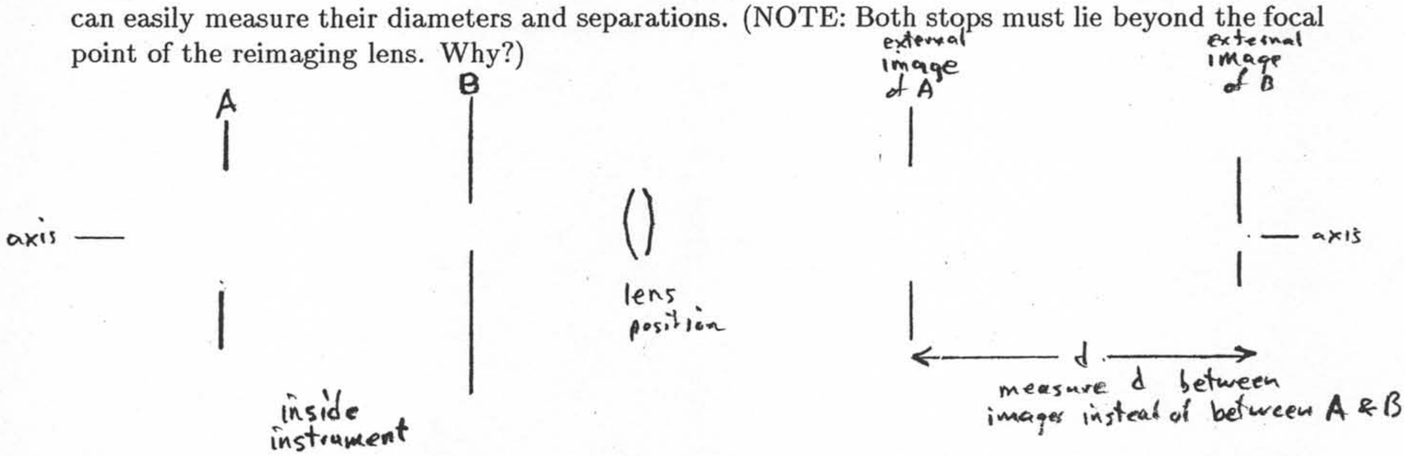
   Remember that the entendue U of an object or source and an image which are a distance r apart is

   $U = s\Omega = \frac{sa}{r^2} = a\omega$

   where the image has area a and subtends a solid angle Ω to the source and
   where the object or source has area s and subtends a solid angle ω to the image.

   ← Try 9 cm f.l., ~13 cm from exit.

3. Fill the instrument with light from the source. Using a lens outside the output of the instrument, image both the final aperture and the processing aperture outside at convenient distances so you can easily measure their diameters and separations. (NOTE: Both stops must lie beyond the focal point of the reimaging lens. Why?)



   Calculate the etendue U for the external images as drawn above. (The value of U should agree with the etendue determined from the internal measurements because the etendue is not changed by a lens that is large enough to accommodate all the rays exiting the instrument.)

4. In an actual coupling problem you would try to choose a lens, or a combination of lenses, so that the stop images would lie on the pupils of the instrument to be coupled to, and have the right magnifications to couple efficiently into it.

4 EXPERIMENTS IN COUPLING THE FLUX

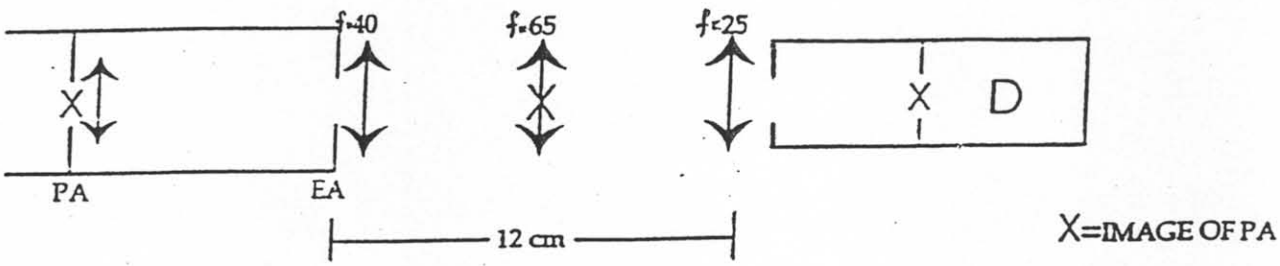
Remember, the system cannot transmit more light than allowed by the components which have the smallest etendue.

"use" $\frac{1}{2}$ - $\frac{2}{3}$ dia of source)

1. Now place the diffused source in front of the nullometer (not too close - ~~remember item 3 on uper page 4 above!)~~ so as to fill the instrument. Couple the sensor to the instrument in one (or both) ways:

   a. Processor aperture imaged (approximately) on outer detector stop; exit aperture on detector. This can be done using 40 and 25 mm lenses respectively.

   b. Processor aperture imaged onto detector surface:



In this diagram, the "instrument" is on the left and the detector is on the right.
In this diagram "X" marks either the PA aperture or its images.
The diagram shows

- the right lens and aperture PA and exit aperture EA of the instrument,
- three added lenses f=40 mm, f=65 mm and f=25 mm
- the detector aperture (diam=9 mm) and phototransistor width.

SKIP for now

5.9 Does the signal S agree with predictions based on separate estimates of instrument's etendue U, transmission τ, detector sensitivity η and source Luminance L?

$$S = U\tau\eta L$$

2. Is the signal independent of source distance? If so, within what range? Keep source on-axis. Is it now possible to add coupling optics between the source and the instrument to increase the signal?

3. Without disturbing the alignment of instrument and detector, move the source 300 mm away from the entrance aperture. Record the signal S. Are you filling the instrument? Verify $1/R^2$ when source is smaller than f.o.v.

OPTIONAL

4. Add a lens to restore the coupling (this can be done in several ways). We call this a "relay lens". Now what signal do you get? Try moving the source out to 500 mm.

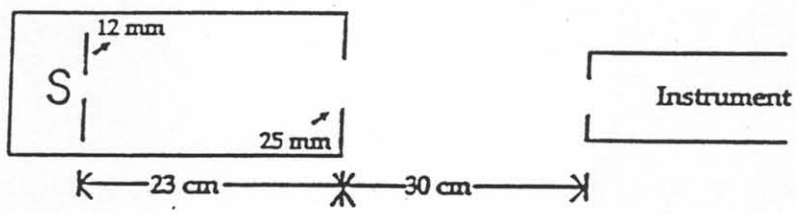
5. With the detector still in place, set up a 6 mm aperture 200 mm in front of the entrance aperture. Put the source (diffused) about 80 mm behind the aperture.

   - Checking visually, is the entrance aperture filled with light?
   - What about the processor aperture?
   - If the system is filled, the signal should agree with that of item 3 on page 6 above. Does it? Wait a minute...whats going on here?

6. Mount a 12 mm aperture directly in front of diffused source, black side toward source. Leave this aperture in place until further notice. Now set source aperture 140 mm from instrument entrance aperture, and measure the signal. Set up a FIELD LENS at the entrance aperture to image the source onto the entrance aperture.

   - Do you have available any lenses of the right focal length? If not, a fast, short focal length lens can be improvised from two similar lenses back to back.
   - Which way should they be oriented? Try it and compare results with the first method.

7. Suppose that the source is now inaccessible, perhaps being located inside a vacuum chamber with a small window. Set the system up as follows:



   Using a single lens, couple to the instrument as best you can. How does the signal compare with the ideal value of S of item 3 page 6 above?

8. Add a field lens to this system, locating it near the entrance aperture. The relay lens used in item 4 of page 7 above may have to be readjusted slightly. What signal do you get now?

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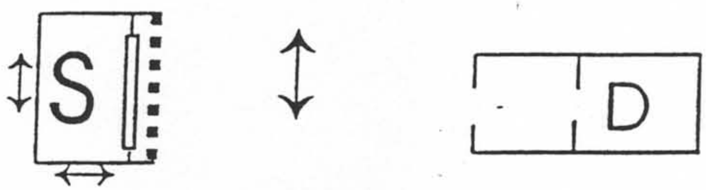
5 DETECTOR / SOURCE EXPERIMENTS : All optional

1. O1 Optional - Do other parts first.
   Investigate spatial uniformity of detector by forming a small (1 mm diam) light spot with a lens and a pair of stops and scanning with the translating stage detector across it. Make sure the 9 mm diam stop of the detector does not limit the signal.

2. Investigate the source spatial uniformity by forming a small virtual detector across which the source may be scanned. The image of thew detector can be placed close to the surface of the diffuse source and so can measure the Luminance of particular small regions of the surface. This method of measuring the luminance is useful if the source cannot be reached. For example, the source may be at a high temperature, may be at a high voltage, may be in a vacuum system or may be remote, perhaps on the moon.

   Also check the angular dependence of the source luminance.



3. Set up the detector 50 mm from black side of a 12 mm stop. Place the diffused source 15 mm behind the stop and measure the signal. AS ALWAYS, OBSTRUCT THE BEAM WITH SOMETHING BLACK to obtain a zero reading and make sure there is no avoidable stray light. Now hang the black shroud over the white side of the 12 mm stop. What happens? Can you figure out why?

   Try moving the source back away from the stop in each case, taking care not to get off center. The signal should, of course, remain constant (up to a point). Does it? What precautions are necessary to ensure that it does? Take care to avoid this photometric problem in later steps.

4. Place the detector assembly near the source, such that the signal S is independent of distance. If L is the Luminance of the source, U is the etendue and η is the sensitivity of the detector, then the signal measured by the detector is

   $S = LU\eta$.

   What is the etendue of U of the detector?
   Determine the sensitivity η of the detector in mA/bulb.

5. Can you improve the signal by use of an appropriate external or "relay" lens arrangement? What is the best you can do?

6. O2 Optional - If time permits.
   The linearity of the detector response could be measured with the help of neutral density filters. Another equally good method, however, is to use an extended source of constant luminance and vary the etendue. Two stops (one can be the detector itself) a variable distance apart will do the trick. Dont forget the zero corrections, if any.

7. Is it going to be possible to couple the entire instrument throughput into the detector? Calculate the detector etendue from the descriptions given in the introduction.

8. O3 Optional

Measure the instrument transmission τ (pretend that the source is monochromatic) by setting up a beam which NOT fill the instrument and can all be put into the detector. Then measure signal with/without instrument in beam. The bright source may help, since the etendue will be low. Be sure no light spills over any stop in the instrument. The detector will have to be moved to different positions for the two measurements. → 5.9 on