

# Physics 625 — Optics

## Experiment 3

### THICK LENSES

Background: Mathematically any two coaxial thin lenses of focal lengths  $f_1$  and  $f_2$  which are spaced by a distance,  $a$ , can be replaced by a "thick lens" of focal length  $F$  and with principal planes at  $b_1$  and  $b_2$  (measured inward from lenses 1, 2 respectively).  $F$ ,  $b_1$ , and  $b_2$  are given by:

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{a}{f_1 f_2}} \quad \text{and} \quad \boxed{\begin{aligned} b_1 &= (a \cdot F / f_2) \\ b_2 &= (a \cdot F / f_1) \end{aligned}} \quad (1)$$

This can obviously be extended to include any number of coaxial thin lenses. For paraxial rays, actual thick glass lenses can be made equivalent to pairs of thin lenses appropriately spaced, so it follows that any coaxial group of actual lenses can, to the first order of approximation, be replaced by a single thick lens model as in Figure 1.

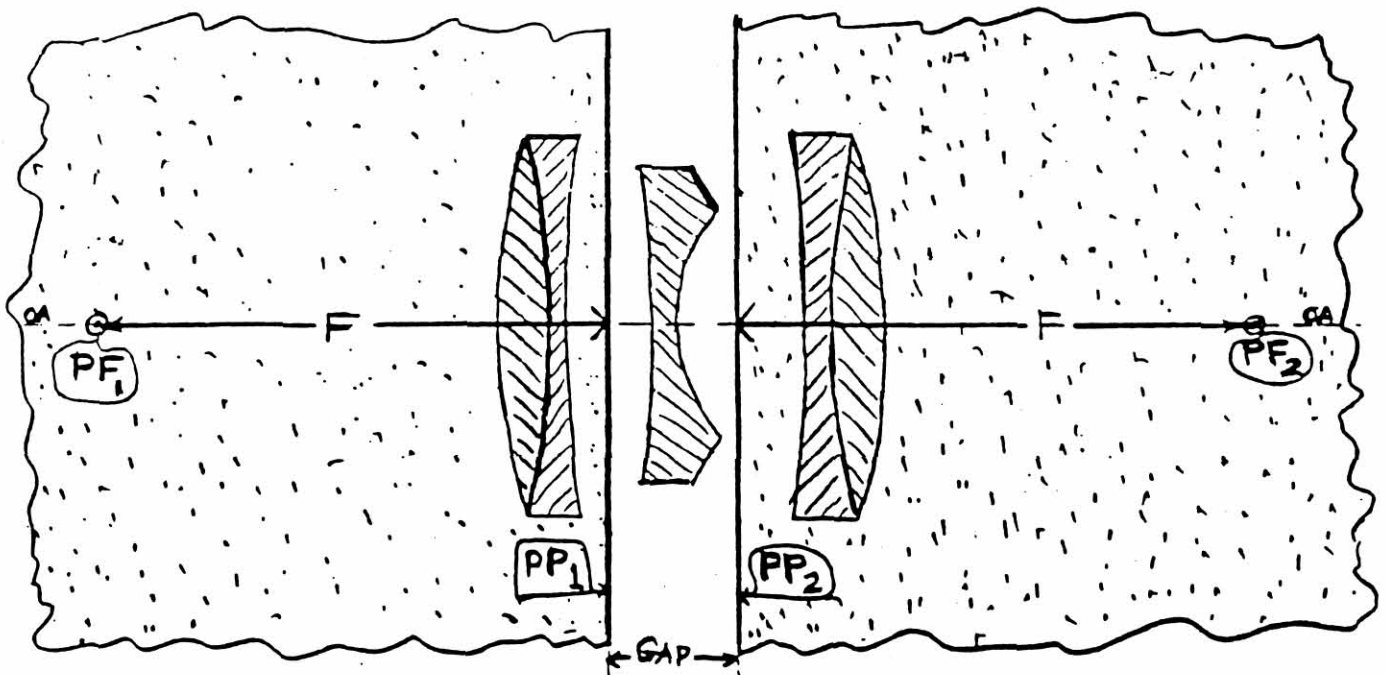
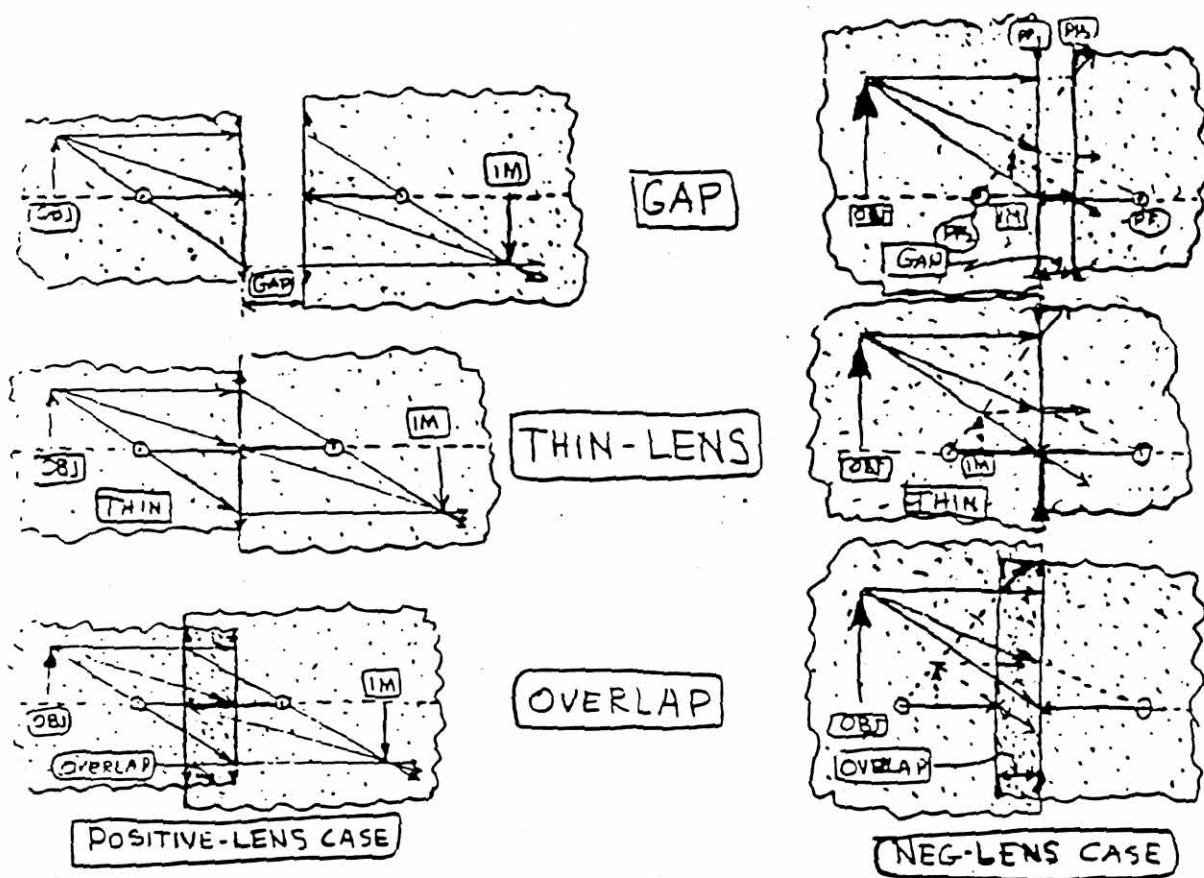


Figure 1. An actual lens showing its focal length, ( $F$ ), its two principal foci ( $PF_1$ ,  $PF_2$ ), and its two principal planes ( $PP_1$ ,  $PP_2$ ).

An easy way to visualize the process of thick-lens ray-tracing is to imagine tracing rays on a thin-lens diagram of the same focal length, then cutting the paper along the principal plane and pasting it over the thick lens diagram so that the cut edges coincide with the two thick-lens principal planes. Note that in some cases the two cut pieces will have a gap, in other cases they will be overlapped, depending on the lens. In Figure 1 they have a gap. Ray diagrams for both types are shown in Figure 2.



The two types of thick lenses.

Figure 2.

**Experiment:** The experiment consists of measuring focal lengths and locating principal planes for some actual lenses. In the case of a high quality lens a large aperture can be employed, but simple lenses should be stopped down to achieve a good compromise between errors due to spherical aberration and errors due to a large depth of focus. Another auxiliary measurement that is pertinent to a high quality thick lens is the determination of the aperture -- the "f/number" -- of the lens.

**Methods:** Position of focal planes can be determined from a source of collimated light, as in Fig. 3.

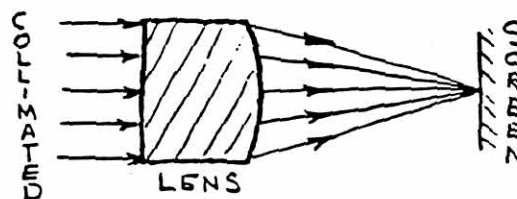


Fig. 3. Finding focal plane using a collimator.

With less equipment, the two focal planes may be located by autocollimation, as in Fig. 4. You measure the "focal distance" -- which is not the "focal length" -- using the inside calipers on a meter stick to find the distance from focal plane to the "lens vertex", as shown.

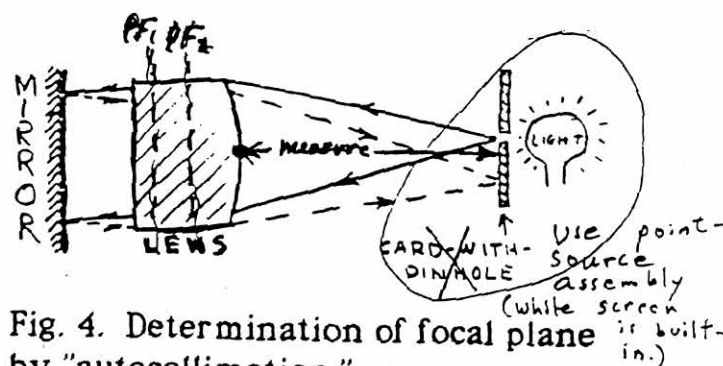


Fig. 4. Determination of focal plane by "autocollimation."

From the thin lens experiment for the case of + lens, real object, and real image,  $d_o/F = |M| = F/d_i$  (see Fig. 5), giving Newton's equation,  $F^2 = d_o d_i$ . From Fig. 2, we can apply this to our thick lens by moving the pinpoint source back (experimental accuracy being best when it is moved by roughly  $F$ ). Now set up a screen to locate the sharpest conjugate image of this source, and measure respective distances to the lens vertices. By subtracting out the previously measured focal distances, you can determine the two distances,  $d_o$  and  $d_i$ . From Newton's equation, the focal length  $F$  is their geometric mean,  $\sqrt{d_o d_i}$ . Once  $F$  is known, you can measure inward from the focal positions by a distance  $F$  to locate the two different principal planes.

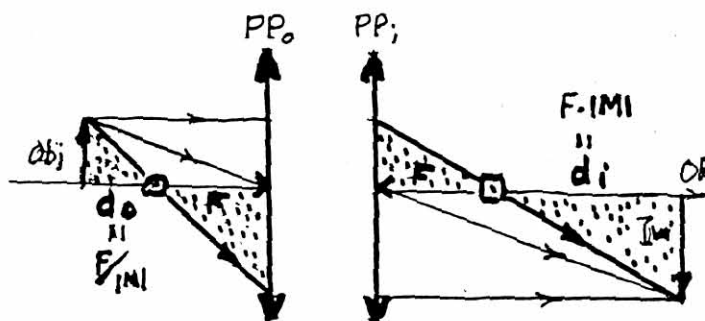


Fig. 5. Principal planes located from the focal distances and the conjugate measurement of  $F$ .

Alternatively, positions of principal planes can be determined by the nodal slide method, i.e. by finding the point along optical axis about which the lens can be physically rotated without motion of the image, as in Fig. 6. (This method is used in the experiment on aberrations.)

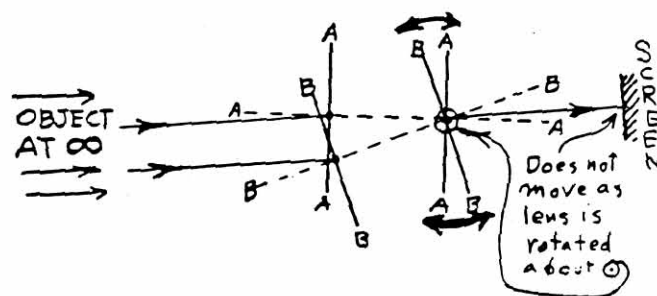


Fig. 6. Location of principal point using a nodal slide. (Ray through PP maintains its direction.)

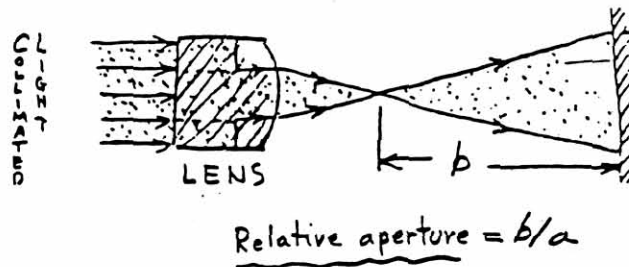


Fig. 7. Relative aperture measurement with a source of collimated light.

Relative aperture, (or  $f/\text{number}$ ) is not directly measurable in complicated lenses, ~~but it can be indirectly measured from the angular size of the emerging cone of light, as in Fig. 7.~~ However it is easily measured from the emerging cone of light when collimated light enters, as in Fig. 7. In fact, as illustrated in Fig. 8, it can even be measured by placing a pinhole at the desired image position and flooding the other side of the lens with diffuse light, then measuring the well-defined cone of light emerging from this pinhole. (Explain why this works.)

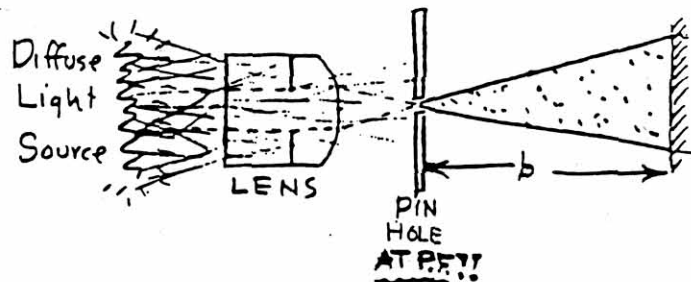


Fig. 8 Using diffuse light and a pinhole to determine the  $f$ -number from an emerging cone of light.

Food for thought: How might you go about measuring the thick-lens parameters of a diverging thick lens?

Now that you know the thick lens parameters of an actual lens, change the object position drastically, for example placing the object three or more focal lengths beyond the object focal point, and carry out a conjugate thick-lens calculation to locate the image. Then set up the source to this position and see how well your calculation locates the measured image. (It is more challenging to do this with a telephoto lens, if you have measured one.) It is recommended that you help visualization by making wiggly lines as in Fig. 2 to simulate the pieces of paper, gapped or overlapped as the case may be. Draw the locator rays right on this sketch.

Poor man's equipment: If time remains, we would like you to discover-by-doing that fancy equipment is not needed for autocollimation measurements. You will then be willing to do it later, right back in your own research laboratory. To improvise a pinhole source, glue Al-foil over a small white card, and poke a pinhole through, in the middle. (The glueing holds the two layers in contact to avoid your having a nasty pinhole-collimator.) Prop this up immediately in front of an ordinary frosted-bulb incandescent light, foil toward the bulb. The white card then serves as the screen to display the returned pinhole image, as in Fig. 9. Now repeat one of your previous sets of measurements and see if it doesn't check out quite satisfactorily.

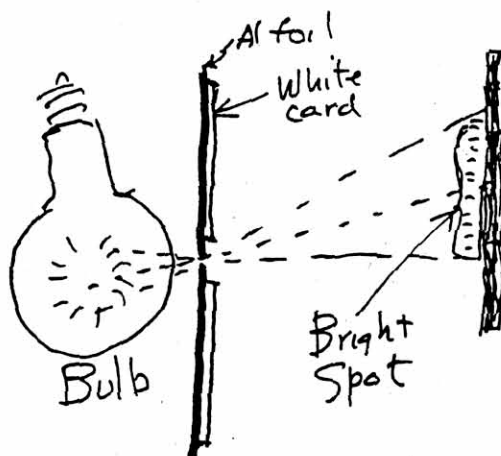


Fig. 9. "Poor man's" autocollimation source.