THIN LENS EXPERIMENT - 2

1 Basic objectives of this experiment.

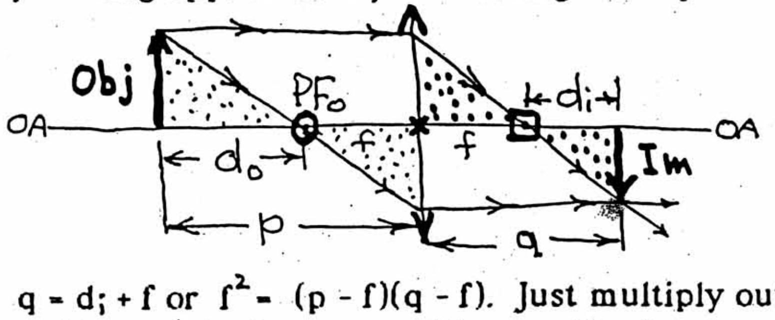
Except for optical engineers and others who work a lot with lens conjugate problems, most people approach such problems with the dusty old lens equation they learned in elementary physics -- something like $1/f = 1/p + 1/q$, with f + *converging* and p,q + *real*. Such summed-reciprocal equations are messy and error-prone with confusing sign conventions.

In this experiment we use two simpler approaches, both based on tracing three locator rays through the two PP's and the OC. We then run through all of the basically different conjugate ranges for both + and − lenses. There are four types of ranges for converging, and four for diverging lenses, the regions being separated by special cases where *magnified* changes to *minified*, *erect* changes to *inverted*, or *real* changes to *virtual*, and in two of which M goes either to zero or infinity. In each example you will first compute M and the conjugate position, noting whether the image is *mag/min, real/vir,* and *erect/inv.* You will then set up the case physically to check it out and thus be sure you get the "feel" for what it's all about.

Begin now by reviewing <u>thin lens terminology and abbreviations</u>. Define each term listed below and check with your partner ... if necessary, check with the book, but <u>first</u> see how many you already know without turning to the book. -- A better way to cement things in your memory.

Optical axis (OA); optical center (OC); principal plane (PP); principal focus (PF); object space, ray or point; image space, ray or point; conjugates; *object/image, erect/inverted; real/virtual*, magnified/minified; magnification M + erect). In the rest of this experiment we will use these specialized words or their abbreviations freely.

Begin by showing yourself that the dusty old lens equation can be derived mathematically from Newton's equation $f^2 = d_o d_i$ (which is based on the ray-tracing approach) by converting d's to p's and q's where



$p = d_o + f$ and $q = d_i + f$ or $f^2 = (p - f)(q - f)$. Just multiply out and do some cancelling. We are playing fast and loose with sign conventions by considering only the simplest case: converging lens, object and image both real. We'll improve on this later.

② Setup for this experiment.

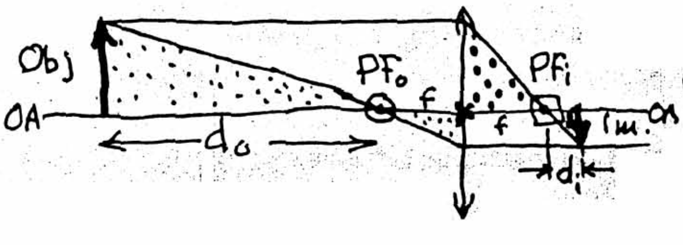
In order to deal smoothly with changing object positions where *virtual* objects or images show up, we use an object that is generated physically right out in the air using a projector. Thus, for example, when a *real* object approaches closer and closer to the PP, we can push it right through the lens plane to become a *virtual* object without any discontinuity of the arrangements. Similarly, we pull an *image* out of the air with an inverted projector so it can move smoothly from a *real* to a *virtual* image. The image is viewed on a small disk of ground glass that is part of the image pick-off and moves with it. To be able to treat an object or an image at infinity, a small [redacted]-mounted negative lens is provided for slipping over the end of either object or image device. (Push it on until it is stopped mechanically.)

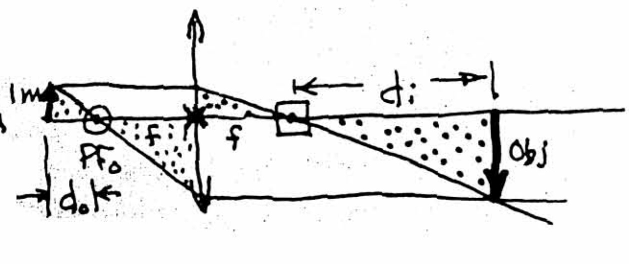
Let's start by locating the position of object and of image out in the air. There is a simple ground glass screen on a slider provided. Move it along the optical bench until its output object is sharply in focus on the screen. Set the end of the object-marking rod to this position.

Now try bringing the image pickoff into a position where this image on the ground glass screen appears sharply in focus on the output screen of the image pickoff device. (Likely it will be too faint to see unless the room is unusually well darkened. A lot of light is scattered away by each ground glass screen.) OK. Just remove the first screen; the object position and size will be unchanged, but the final brightness will be much improved. Mark the image position by the end of the pick-off device rod.

You will see both an object reticle and a scale on the final pickoff screen. Comparing the two, you will see the scale measurement at <u>unity magnification</u>. Note these M = 1 readings on the scale.

The two simplest ranges for calculating conjugate positions are those in which both object and image are *real* and the lens is *converging*. About 90% of the cases one is likely to encounter in practice are in these two categories (one range being *magnifying*, the other *minified*). All of these can be solved *in your head* if the actual numbers are not too complicated for head-calculations.





(②- continued.)

From the two similar triangles that touch at PF_o, we see that $|M| = f/d_o$, and from the two that touch at PF_i, $|M| = d_i/f$; both together yield Newton's equation, $d_i/f = f/d_o$. For example, if you want to focus a camera lens for an object placed **20**·f ahead of PF, the camera film must be adjusted to (**1/20**)·f beyond PF (i.e. beyond the film position that is perfectly in focus for objects at infinity). If you want |M|, it is simply f/d_o. Notice carefully that d_o and d_i are measured, not from the PP but from PF_o and PF_i, respectively, which is the reason the equations are simple ratios, not the dusty old sums of reciprocals. This was Newton's approach to conjugate problems, but the ratio/inverse-ratio method used in the **20**·f vs. (**1/20**)·f example above is an easier thing to remember and carry through in your head. Notice the use of subscripts and round or square symbols to distinguish the two kinds of principal foci, PF_o and PF_i, especially valuable in keeping straight the ray-tracing steps for negative lenses.

We hope that this experiment will persuade you to adopt these more practical methods of conjugate calculation for the future -- you have lots of years ahead of you to profit from them. (A second, completely general method will be given you in a few minutes.) Whether or not we succeed in this long-run conversion of you personally, we **do** expect that you will stick strictly to our shorthand methods at least for the duration of this experiment.

FAIR ENOUGH?

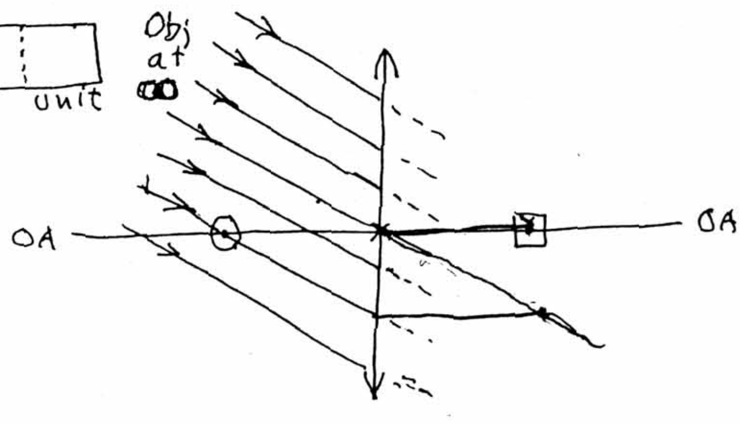
(3) Measure the focal length of our converging lens

When the object is at infinity, the image falls, by definition, on the focal plane (FP_i) and thus it locates the image-principal-focus (PF_i). A simple lens behaves as a thick lens, with two principal planes (PP) that are separated by roughly 1/3 of the optical-axis-thickness of the simple lens. For this experiment we can neglect this relatively small correction, deliberately sacrificing any claims for great precision in whatever we do and measure today, our objective being merely to familiarize you with methods and general patterns before getting into the thick-lens refinement.

To measure the focal length f of our converging lens we generate an object at infinity by slipping the brass-mounted conversion lens over the object projector. Now by moving the ground glass screen back and forth along the bench you can find the position of sharpest focus which is on the focal plane. With inside calipers you then measure the "**focal distance**" between the lens surface and the ground glass, treating it as the "**focal length**" which is not quite correct. (If you prefer to be a little more precise you can cheat a little by putting in a result of the thick lens refinement, simply adding to the focal distance 1/3 of the lens thickness. Take your choice and write it down here for future use the value

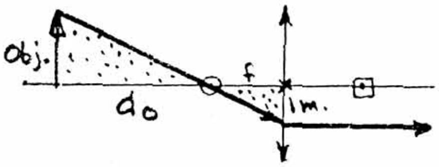
{MEASURED} f = + ______ unit

Draw qualitatively two locator rays for this image. What happened to the third ray?

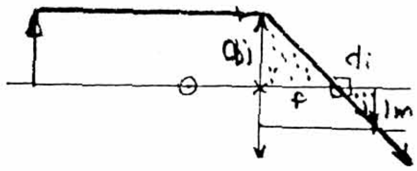


④ + lens, real object, real image, minified.

Set up real image at distance 2f outside of PF0 (or 3f ahead of PP).



$|M| = \frac{\text{Im. size}}{\text{Obj. size}} = \frac{f}{d_o} =$ ☐



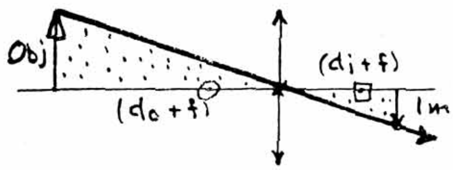
$|M| = \frac{d_i}{f}$ or $d_i =$ ☐

Check??

Now check:

$(d_o + f) =$ ______

$(d_i + f) =$ ______



Ratio $\frac{d_i + f}{d_o + f} = |M| =$ ☐

Image is [magn'd/min'd] [real/vir] [erect/inv]

What is M, sign considered? ☐

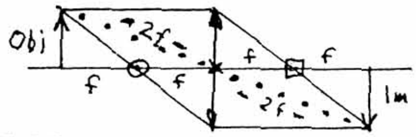
Find the image. (di + f) measures ☐

Read off the image.

Size from reticles: M measures ☐

Do your measurements check with your calculations within reasonable experimental error? [Yes/no]

Special case: Unit magnification, 2f/2f.



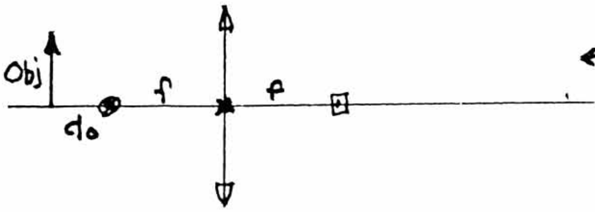
Study the picture; don't bother to set it up.

Now go the in-your-head route: for top of page example:

$d_o/f = |M|^{-1} =$ ☐, so $|M| =$ ☐, $d_i = f|M| =$ ☐

⑤ + lens, real object, real image, magnified.

Put object 1.5 f beyond lens (i.e. 0.5 f beyond PFo).



← Draw all three locator rays, freehand.

Distance do/f = [ ] = 1/|M|

|M| = [ ]

di = Mf = [ ]

Bring image into focus. Physically.

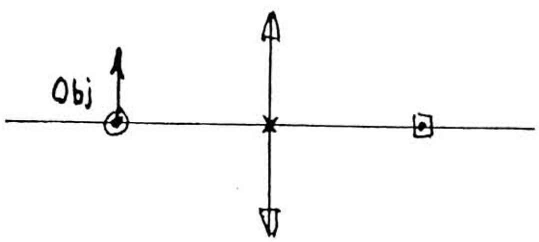
di measures [ ]

M measures [ ]

Image is [mag/min], [real/vir], [erect/inv]

Notice that this represents previous example with object and image interchanged.

Special case: + lens, real object at PFo

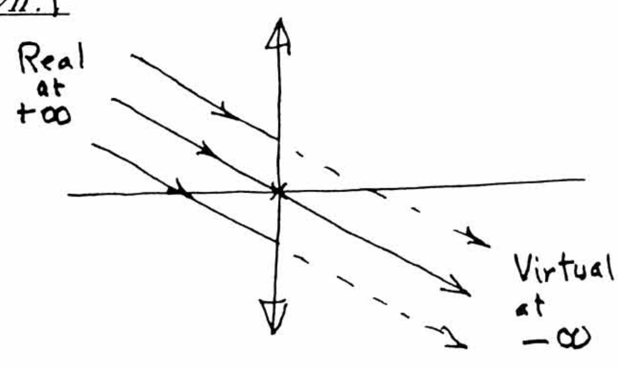


Draw two rays. (Where's the third ray?)

|M| = [ ]

Image is [erect/inv], [mag/min],

[real/vir.]

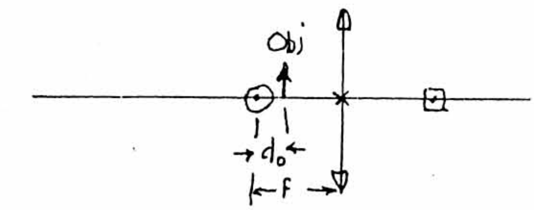


Oh, oh. You can't cross out either real or virtual; could be either.
(This is special case where
real ↔ *virtual*.)
Set this up if you like, slipping brass infinity-producing lens onto receiver.
(Now remove it again.)

Review: From object real at infinity thru 2f/2f to object at PFo, M goes down steadily from 0 to −∞, (i.e. |M| increases). At first *minified*, past 2f/2f case *magnified*. Image always *real, inverted*,

⑥ + lens, real object less than f from PP.

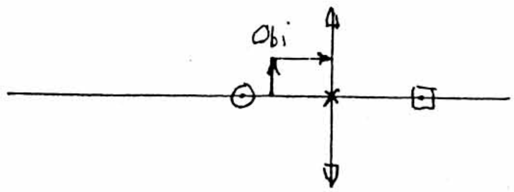
Put object inside PF0 by f/3 (i.e. 2f/3 from PP).



|M| = [ ] / [ ] = [ ]

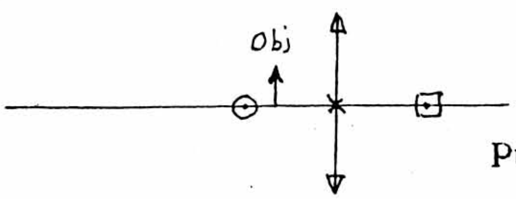
Erect/inv mag/min real/vir

M = [ ] (+erect, −inverted)



di/f = [ ] / [ ] = M = [ ]

Image real/vir ?



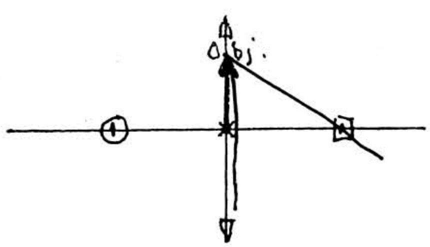
Im-to-PP dist. / Obj-to-PP dist. = [ ] / [ ] = [ ]

Check?

Put eye here and you have an ordinary "magnifying glass."

Now find image physically. Real/vir at [ ] from lens.
M measures [ ] erect/inv? Note: ⊔⊔ inverts to ⊓⊓.

Special case: Object and image lie in PP.



Draw locator rays. Where is third ray?

Image is at [ ] from PP.

M = [ ]

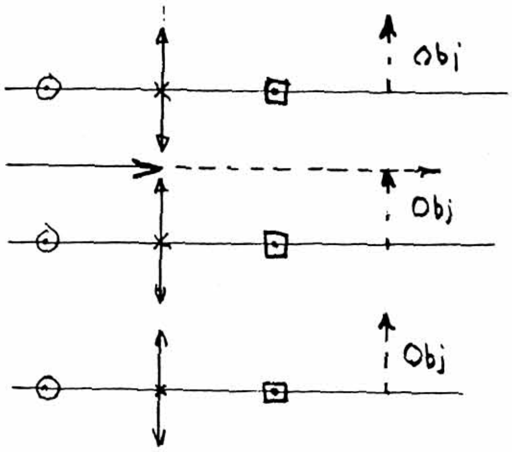
Image is erect/inv mag/min real/vir?

Well! *Mag/min* came out neither; M = 1; *real/vir* came out either.
It's a special case where *mag* ↔ *min*, and for object *real* ↔ *virtual*, while for image *vir* ↔ *real*.

Review: Everywhere *magnified, erect.* As object moves from PP-distance f to 0, M proceeds from +∞ down to 1.

(7) + lens, object virtual.

Set up object at 2f inside lens.



Draw PF_o ray

M = [ ] / [ ] = [ ]

Draw PF_i ray to final image.

$d_i = M \cdot f$ = [sign |]

(+ sign of f for converging lens.)

Check:

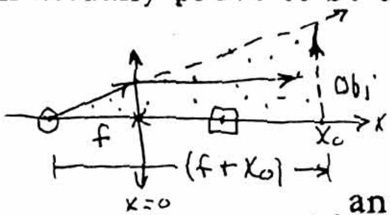
M = Im.-to-PP dist / Obj-to-PP dist = [ ] / [ ] = [ ]

Locate image physically and compare position, magnification.

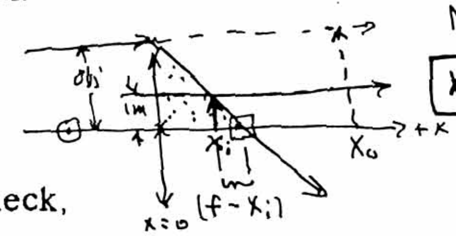
Review: For all cases of + lens, *virtual* object, image progresses only from PP to PF_i as object goes from PP to ∞. Thus M, always +, goes from 1 to $+\infty$. This special case of + lens analysis closes back on the starting point.

Our other conjugate method -- completely general.

This is the best place to develop our general formulation applicable to both + and - lenses, *real* and *virtual* objects and images. Let us define an x axis, its origin at the OC, and taken + in the direction that light is moving through the lens. *Thus all x's are measured, not from the two PF's, but from the PP of lens.* The object distance will be subscripted x_o, and the image distance x_i. For converging lens, f will be +, diverging -. This region: + lens, x_o and x_i both +, (all +) simplifies consideration of signs in deriving our relationship, which will actually prove to be completely general.



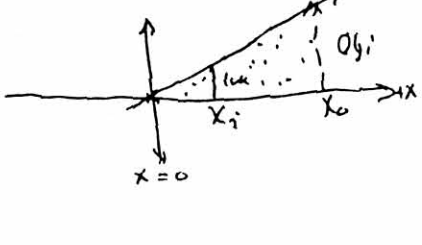
$$M = \frac{f}{(f+x_o)}$$



$$M = \frac{f-x_i}{f} \text{ or}$$

$$x_i = f(1-M)$$

and obviously as check,



$$M = x_i/x_o$$

(8) Apply these to foregoing + lens case, where x_o was 2f.

(1) $M = \frac{f}{f + x_o} = \frac{[\ ]}{[\ ]} = [\ ]$

(2) $x_i = f(1 - M) = [\ ] \cdot (1 - [\ ]) = [\ ]$

Check earlier result?

(3) Check: $M = x_i / x_o = [\ ] / [\ ] = [\ ]$ Check?

All those sign reminders: [ ] may seem a little foolish with everything + in this case, but in the more general cases to which this also applies, <u>keeping careful track of signs</u> is the main thing you have to be careful about. If you do that, all possible conjugate problems reduce to "duck soup."

Let us go back to the previous + lens case of *real* object and *virtual* image as an exercise. Here x_o is negative, being f/3 lens-ward of PF_o

or x_o = [−2/3 f]

Then $M = \frac{f}{f + x_o} = \frac{[\ ]}{[\ \ + \ \ ]} = [\ ]$ Check earlier result?

$x_i = f(1 - M) = [\ ] \cdot (1 - [\ ]) = [\ ] \cdot [\ ] = [\ ]$ Check earlier?

Check $M = x_i / x_o = [\ ] / [\ ] = [\ ]$??

On your own, check (4) where object was 2f outside of PF_o

$M = \frac{f}{f + x_o} = [\ ]$, $(1 - M) = [\ ]$, $x_i = f(1 - M) = [\ ]$

Check: $M = x_i / x_o = [\ ]$

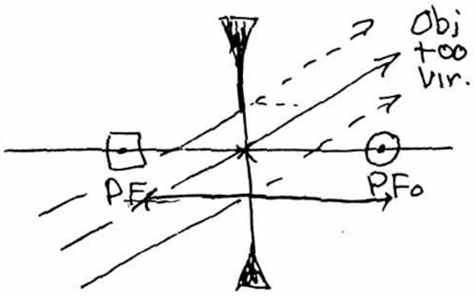
(<u>Note</u>: *minus* M for inverted image.)

9 Repeating everything for negative lens conjugates.

Analogy between + and - lens conjugate regions. Let us multiply f and every x by -1 in our three equations. M is unchanged either in magnitude or sign! This means that for a - lens, with all *real* objects or images changed to *virtual* -- and conversely -- we get the same conjugate patterns that we had for the + lens.
To bring out this analogy, we will just go through all our preceeding + lens examples in the same order but with *real*↔ *virtual* on each object, finding that for each image *real*↔ *virtual* in turn.

- lens, virtual object at OO.

Put OO correcting lens onto projector to give object at OO (*real* or *virtual* are the same at OO.)



Draw rays, inverting positions of PF_o and PF_i for all negative lenses.
As before, one ray is unusable in practice. Take x_o as +OO so f/(f + OO) becomes 0 and (1-M) is +1.
Hence x_i = (1)f but since f is now negative, x_i is - |f|, and falls on PF_i, making it a *virtual* image. Bring the image on receiver into focus and check this.
Aha! At the same time we have determined the focal length for our negative lens. Measure this, correcting as before, if you wish, for the thick lens effect. (Specifically, add 1/3 the thickness of the lens at OA to the measured focal distance, taken +, then change the sign of the combined result. For the rest of this experiment, this is the value you will use for f of the negative test lens.

f = [-]

(10) Negative lens, *virtual* object, *virtual* image.

For analogy with our first + lens case, we will push a *virtual* object through the lens and 2f beyond PF_o (or 3f beyond PP).

f = ______ x_o = ______ $(f + x_o)$ = ______ + ______ = ______

$M = f/(f + x_o)$ = ______ / ______ = ______

$(1 - M) = (1 -$ ______ $)$ = ______ $x_i = f(1 - M)$ =

______ × ______ = ______ Check: $M = x_i/x_o$ =

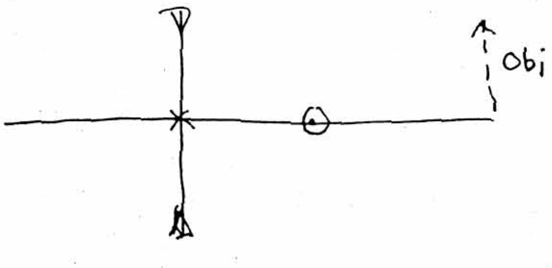
______ / ______ = ______

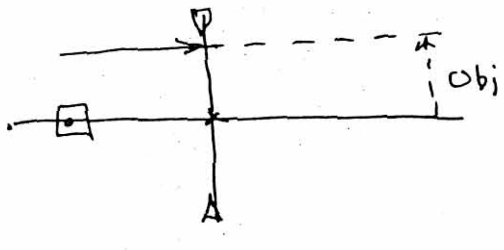
Image is *mag/min erect/inv real/vir.*

If you have kept your signs straight, this should be the exact counterpart of the first + lens example, with *real* ↔ *virtual*

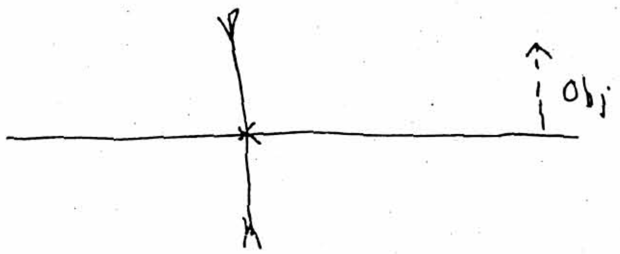
Check back to match the earlier values, remembering that you are now measuring from PP instead of the PF's. It checks? *Yes/no.*

Now move the receiver to find the sharp image. Measured x_i is ______ and measured M is ______ Now, as a final clincher for your understanding, trace all of the rays for these *virtual* objects and images.

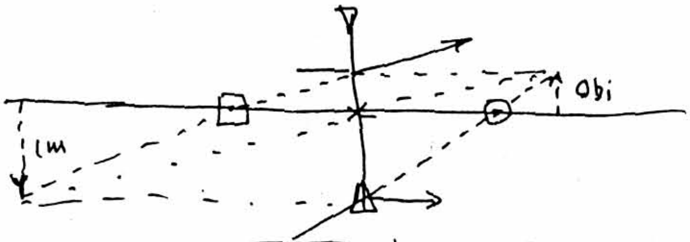




Check



(11) Negative lens, *virtual* object, magnified.



$M = f/(f + x_0)$

$= \dfrac{-|f|}{-|f| + 1.5|f|}$

$= \dfrac{1}{1-1.5} = \boxed{-2}$

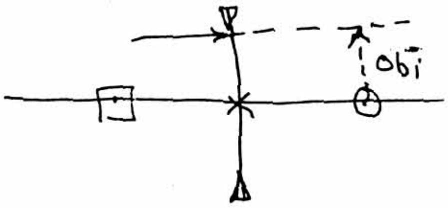
Image [*mag/min*] [*real/virtual*] [*erect/inverted*] $(1-M) = +3$

$x_i = f(1-M) = -|f| \cdot (+3) = \boxed{-3|f|}$

Check.

Review. Images all *inverted* and *virtual*. As object progresses from $x_0 = +\infty$ to $+f$, magnification progresses from 0 to $-\infty$.

Special case: - lens, object *virtual* at $x_0 = +|f|$.



← Draw rays. If there is going to be time, set up object *virtual* at $|f|$ behind lens and observe image with 00 adapter in place on receiver.

M = [ ] (It's boundary between +M and -M regions.)

(12) Negative lens. *virtual* object < f from PP.

Set up *virtual* object less than (2/3) f from PP of lens.

Draw all three rays on the same diagram this time, preferably using three colors of pencil to keep them straight.

$M = \frac{f}{(f+x_o)} = \frac{[\ \ ]}{[\ \ +\ \ ]} = [\ \ ]$

$(1 - M) = 1 - [\ \ ] = [\ \ ]$

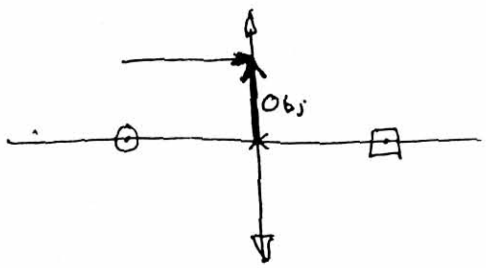
$x_i = f(1 - M) = [\ \ ]$

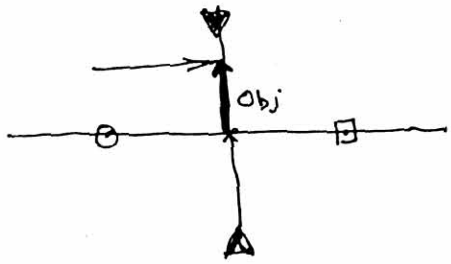
Check: $M = x_i / x_o = [\ \ ]$

Image *mag/min* *erect/inv* *real/vir.*

Special case: Negative lens, object on PP of lens.

Just as for + lens, image falls on top of object. Make a comparison of locator rays for + and − lenses, noting that in both cases only two rays are available to trace.





(13) Negative lens, object *real*.

Set up *real* object 2f in front of PP of lens.

f = [ ], x_o = [ ], $(f + x_o)$ = [ ],

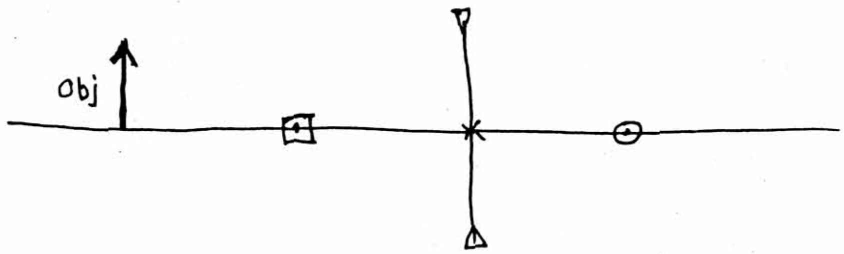
$M = f/(f + x_o)$ = [ ], $(1 - M)$ = [ ],

$x_i = f(1 - M)$ = [ ],

Check: $M = x_i / x_o$ = [ ].

Image [*mag/min*] [*erect/inv*] [*real/vir*]

Draw all 3 rays using colored pencils if available.



Review: As *real* object progresses from $x_o = 0$ to $-\infty$, image progresses from +1 to 0. Image always *virtual, erect, minified*.