KEY TOOLS IN FOURIER IMAGE-PROCESSING EXPERIMENT

Physics 625 -- 92/4/24 1998 March grouping

(We consider here only perfectly monochromatic light.)

Good reference: All of part 6 in Miller & Roesler

1 - Repetitious object gives **sharp** diffraction pattern. Woven-wire object -- essentially two diffraction gratings at right angles -- gives "cornfield pattern" of equally-spaced **sharp** points of light. The **sharpness** comes from the extensive **repetition** of the object.

2 - Big ⟷ small. **Small-scale** aspects of a pattern show up as **large-scale** aspects of its Fourier transform. A major purpose of this experiment is to provide experience -- in a variety of contexts -- with this seemingly contradictory principle.

3 - Babinet's principle (for complementary obstacles). Given an **obstacle** of various clear and opaque regions, its **complement** consists of an interchange of all the clear and opaque regions. Example: a hole drilled in a tin plate is the complement of a black dot of the same diameter and position as the hole.

In one position, **A**, in space we will place the **obstacle** or its complement, on another surface, **B**, we will observe the **pattern of light** that has passed through **A**.

(a) Consider any position on **B** which is **dark** with **nothing** (neither obstacle nor complement) in the position **A**.

(b) If this same position on **B** becomes **bright** when the **obstacle** is put in place on **A**, it will be **equally bright** when the **complement** is substituted for the **obstacle** on **A**.

Take a few minutes to work out how this principle is simply a consequence of the much more general **Huygens-Fresnel principle**. What about the **phase** of the light in the pattern generated from the obstacle, compared to that generated from the complement?

4 - Conservation of energy. All the **energy** that passes through an obstacle shows up in the resulting diffraction pattern. However, think of Thomas Young's two-pinhole diffraction fringes. Considering a position at the center of one of the **dark** fringes, if we close up either one of the pinholes this position will become bright. Two brights give a dark -- what kind of energy conservation is that?!! A historical dilemma . . . Get the answer very clear in your mind.

5 - Fraunhofer diffraction pattern. Fraunhofer conditions stipulate collimated light arriving at an obstacle which generates a diffraction pattern observed at infinity. One plane wave in -- an infinite set of plane waves out . . .

With the help of a lens, we can take a pinhole source of light and form a point image of it upon a screen, somewhere. When an obstacle is interposed before this image, the **Fraunhofer diffraction pattern** is displayed **at a finite size** on the screen.

You should be able to use the trick of **paired imaginary lenses in contact** to show why this pattern is (except for phase) exactly the Fraunhofer diffraction pattern of that obstacle.

It is important to get these tools straight in your head before coming in to lab for this challenging experiment. It will help you get more out of the exercise; speed things up; carry you farther by the time the period runs out.

DO IT NOW!!

F.I.P. EXPERIMENT -- SUCCESSIVE STEPS OUTLINED

Physics 625 92/4/24 -EEM

I - Set up and align system. (See sketch at end.)

A - Without lenses, track laser pencil beam thru centers of the two screen positions.

B - Add spatial filter, pinhole removed. Center the (now blob) of light.

C - Add pinhole and optimize its focus and centering.

D - Add camera lens and focus pinhole, centered, on masking surface.

E - Add achromat and refocus camera lens slightly. Adjust achromat laterally to center the blob of light on final image-viewing screen.

F - Install the wire-screen object and focus it sharply on final screen. This must be accurate, and final adjustments can best be made by adjusting the achromat while watching the final image with a loupe.

There is a strange problem encountered in trying to focus the wire-screen object on the final screen -- the appearance of a whole series of false-focus positions. It is a sort of vernier-like effect resulting from the combination of monochromatic light and a completely repetitious object. W used to locate the one correct focus out of the set by using white light. Today we make the determination more simply by paying attention to the images of bits of dirt clinging to the screen. Only for the correct focus position do they even show up. Why is that?

II - Experiments using fine, woven-wire screens as object.

A - Masking out or phase-shifting portions of F.T. pattern.

(a) Stare a bit at the elegant F.T. "cornfield" pattern.

(b) Mask off all but **center vertical line** of cornfield dots. (Cut your slit roughly as wide as the bright-spot spacing.) What do you expect? *Try.* Repeat with horizontal line of dots.

(c) Mask out all but a **45 degree line** of bright dots containing the central dot. But first, say what you expect to see in the final image. How about **image spacings**? *Try.*

(d) With crossed slits, mask off all but the **center bright spot**. What do you expect? *Try.* How **bright** is the final screen image (referenced to same image when object is not in place)? Don't **actually** remove object, or you'll just have to readjust it -- just **argue** over this question. What about the **dirt spots**? (Think of **Babinet's** principle.)

(e) **Narrow** one of your crossing slits, to a mm or so and predict the result for the **dirt-spot images.** (How can you get this much effect by just masking away some small regions that appeared to be dark.)

(f) Mask away **center bright spot alone** and try to argue out what you expect as the final image **before you look at it.** Hint: **Babinet's principle** is again useful in figuring out certain aspects. (Use loupe magnifier when you look at the final image.)

(g) **Very carefully** install glass-chip **phase-shifter** over center bright spot, and rotate the chip delicately to get various phase-shifts. Again, **argue this out** before peeking at the final image. **Conservation of energy** will help explain some details of the final image.

B - Revolve the screen about a vertical axis.

(a) Notice the "half-harmonics" that now appear in the Fourier image. How in the world can such apparent fractional harmonics occur? What do they tell you about three-dimensional aspects of this object?

(b) To bring in a large range of higher-order detail in the transformed image, slide the screen near to the achromat and then explain what you then see on rotating the screen a bit each way again.

(c) Now slide the screen all the way to the vicinity of the camera lens. Can you work out a quantitative scheme for relating the **magnification** of the Fourier pattern to the **placement** of the wire-screen object?

C - Another wire-screen with different parameters.

(a) **Looking only at the Fourier pattern from the second wire-screen** (placed approximately in the original position of the first wire-screen), can you make some qualitative statements about the two basic parameters -- wire **spacing**, and wire **diameter** as compared to the first wire-screen?

(b) Still not looking at the wire-screen or its final image, **argue out** how one could determine the wire-size compared to wire spacing if we could meter the **brightness of the center spot** of the Fourier pattern compared to the same, with the wire-screen removed. (We are not set up to meter these, however.)

(c) Still avoiding peeking, figure out a way of quantifying this size-to-space ratio with something you **can** measure from the Fourier pattern. What you can measure is the number of bright spots along the x (or y) axis before reaching the first **brightness minimum** along these lines of dots. You can make good use of both Babinet and Huygens-Fresnel in arguing this out properly!

(d) Now you certainly deserve a peek. (You will probably have to touch up the focus of the achromat a bit, having moved the object slider.)

D - Large scale features of object related to small features of transform.

(a) Again slide the object up close to the camera lens for magnifying small features of F.T. image. Now cover the object with a slit (set at some angle) to constitute the overall aperture, but **first argue out** what you will expect to see as the Fourier image. Now look at the image. Finally, widen or narrow the slit while watching the transform image.

(b) Use two slits at right angles to give a **rectangular** or **square** aperture for the object. Look at the result; this is a rather important image to stare at a little; you will see it fairly often. What the heck! Let's also try the slits at some other angle like 45 degrees.

(c) Now try a **round hole aperture**, the F.T. of which is called an "**Airy disk**." Pretty, isn't it? If you move the round aperture right-left, up-down, or any-old-way, the size, shape of position of the Airy disks won't change. Yet the light "knows" you have been moving the hole around. If you could find and focus the final image of the hole (not easy here because it's close and it's small -- just take our word) you would see that image moving around, all right. What is the information the light in the Airy disks has available that you are not able to detect?

(d) Well, heck; let's check out your answer to (c). Use as aperture **two** holes of identical diameter, but spaced a short distance apart. Look at the resulting Airy disks with a loupe and you will see something beautiful. From this F.T. pattern you can compare the spacing of the two holes with their diameter, quantitatively. Argue it out. Now directly measure both the diameter of the holes and their spacing with a mm ruler.

III - Diffraction gratings.

A - Ronchi ruling. (Grating with <u>clear widths</u> equal to <u>opaque widths</u>).

(a) With rulings running **up and down**, what kind of Fourier pattern are you going to see? Now insert grating as object, and see if you had it right. Why is this pattern different from the sort of grating pattern most people carry around in their head? (Think of how the usual spectroscope is constructed -- as compared to our setup.)

(b) Put in two gratings set at right angles to each other, and compare the F.T. pattern to the woven-wire screens we were using earlier. Try also some angle other than 90 degrees.

(c) The Ronchi ruling has the special property that only the **odd** orders show up, all the **even** orders are missing. **Don't try this yet,** argue it out first, using the Huygens-Fresnel principle.

(d) OK, try it out. Wups. What's wrong? **All** the orders are there. So take a loupe and check the Ronchi. Sure does **look** like equal clear and opaque spaces. Well, in fact they **are** equal. This is pretty interesting: These rulings were reproduced photographically in ordinary gelatine emulsions. The opaque regions are both gelatine and silver, with more total volume than the gelatine alone in the clear spaces. On drying, the clear gelatine sags down between the thicker silver regions producing, when dry, a diverging cylindrical lens of dry, transparent gelatine in each clear space. This cylindrical lens really messes up the phases for each little sub-line of the clear spaces, twisting up the phasor "curly vectors" into snail-like spirals that won't end up at zero for **any** angle of tilt.

(e) We also have available the same Ronchi rulings fitted with an "**optical gate**". The emulsion is covered with a flat glass covering-plate, the space between emulsion and glass plate being filled with mineral oil of refractive index about 1.5, fairly close to that of the of dry photographic gelatine. This cancels out the cylindrical lens fairly well, leaving a Ronchi ruling that works reasonably well. *Try it.* You'll see the even orders pretty well suppressed for the first few even orders.

Incidentally, the order **zero** is **even**. Why isn't **it** suppressed too?

(f) The best Ronchi rulings consist of thin layers of metal evaporated on glass. They're pretty expensive but maybe we can dredge one up for comparison.

B - Line-of-dots grating.

(a) By masking off a coarse grating with two opaque tapes spaced close together we can construct a line-of-dots grating. Think what kind of F.T. pattern that should produce, if the line of dots runs right to left.

(b) OK, now *try it out.* It looks a little like the imression people tend to carry in their minds for an ordinary grating -- remember III A (a)? The **reason** is, of course, different.

IV - Fresnel diffraction.

A - Unfocused Fraunhofer patterns.

(a) Think of the original, beautiful, cornfield patterns of our woven-wire screens. Well, take a moment to restore one of them in position. The cornfield points **are** beautifully sharp and crisp. Now move a white card back and forth anywhere else in the beam and you see an uninspiring mess. You could say all of these messes are "**out of focus**." But in fact they are all also "**Fresnel diffraction** patterns," their much-more-difficult analysis having been worked out in his youth by Augustin Fresnel. The elegant Faunhofer patterns (Fourier transforms) are the rarities or exceptions; the "messy" Fresnel diffraction patterns being statistically far the more common.

(b) We have a glass slide carrying a tiny steel ball-bearing. Put it in the position of our original wire-screen object and you should see it as a black disk, sharply focused on the final screen.

Now I have a nice little story to tell you -- a true story. When Augustin Fresnel was a very young man in France, it was common to have science contests. For his entry, the young Fresnel submitted an essay on diffraction. In those days, a century after Newton's death, hardly anyone else (except Thomas Young in England) believed in the wave model for light. But Fresnel submitted the whole, difficult theory of what we now call Fresnel diffraction. One of the judges was the famous physicist, Poisson, who concluded that although the mathematics and reasoning were brilliant and he couldn't find the flaw, there had to be **something** wrong, because -- *look here*! Poisson argued that following Fresnel's arguments, he came out with the ridiculous conclusion that in the very center of the shadow behind an opaque disk, there would be **light** -- in fact light exactly as bright as if there were no opaque disk present at all! Obviously some fatal flaw somewhere. Well they tried it out, and there -- GOOD GOD -- **was** a little bright spot of light in the exact center of the shadow of the disk! Today we call that bright spot -- you guessed it -- "the **Poisson spot**!" Young Fresnel got his prize, of course.

OK, now move the ball bearing out of focus a bit -- forward or back -- and you, too, will see the same spot. You will also see series of concentric rings of light, analytically not so trivial to calculate, but still obtainable from Fresnel's formulation. You might want to try predicting just the Poisson spot -- without mathematical equations -- from a qualitative Fresnel zone argument.

V - Some games with patterns and their F.T.'s.

A - Statistical information from study of F.T. patterns.

(a) We have a number of kinds of patterns which can be inserted in the position of our first woven-wire screens so that **after** you try to characterize them from study of their F.T.'s on the first screen, you can remove a light block following it to reveal the actual pattern imaged on the final screen. -- A very instructive kind of game if you have any time left.

Historically, large blocks of Physics and Chemistry were worked out by this scheme because only diffraction patterns (often X-ray) were available

(b) Either your instructor or a partner can set up one of these objects, then everybody argues them out -- not necessarily agreeing -- and at the end everybody takes a peek. Thereupon some may gloat and others blush. Good fun.

B - Some clues to keep in mind.

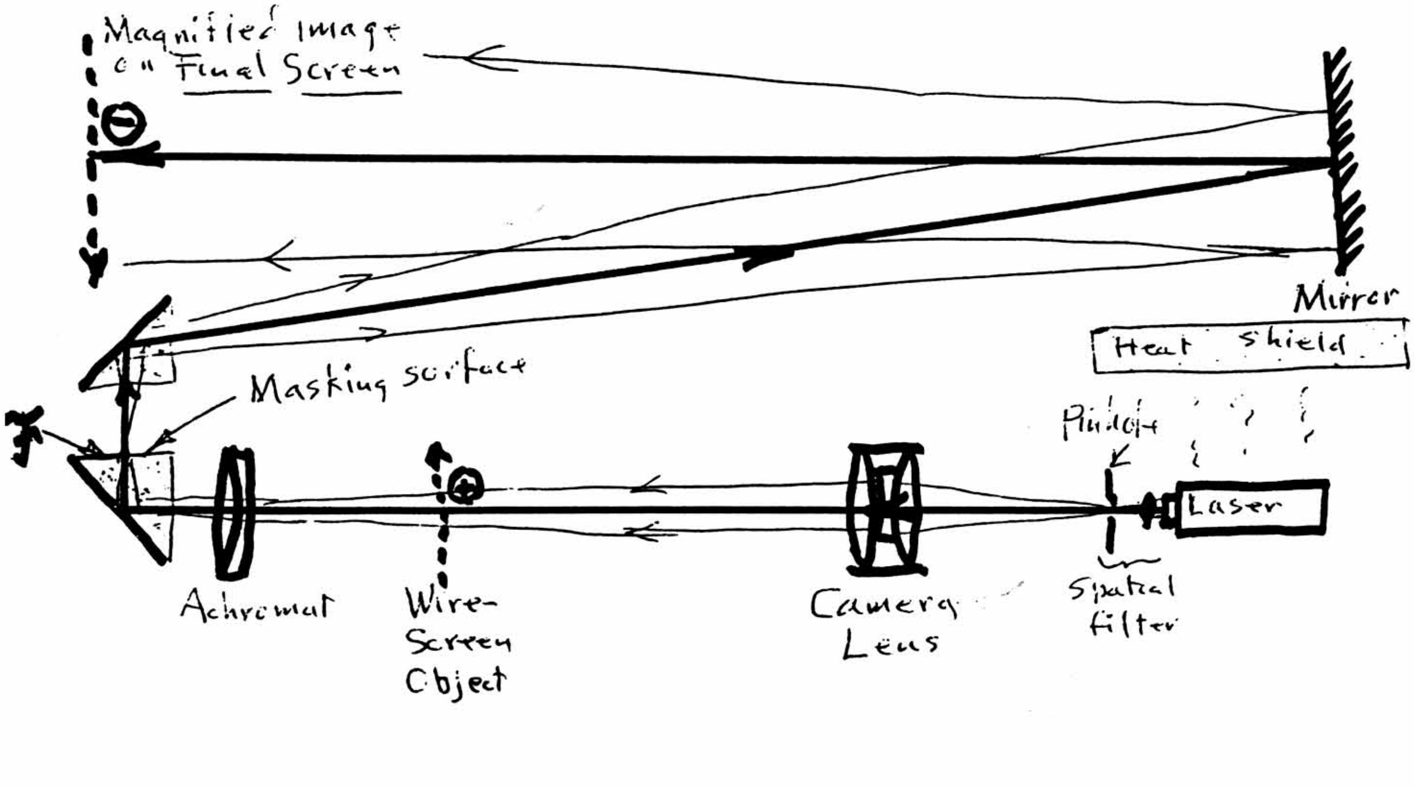
(a) Most of the objects we have, come as a pair of complements. Babinet's principle tells us that (ideally) the F.T.'s for complements should be exactly alike, **except for the central bright spot**, to which B.P. is not applicable. *Why not?*

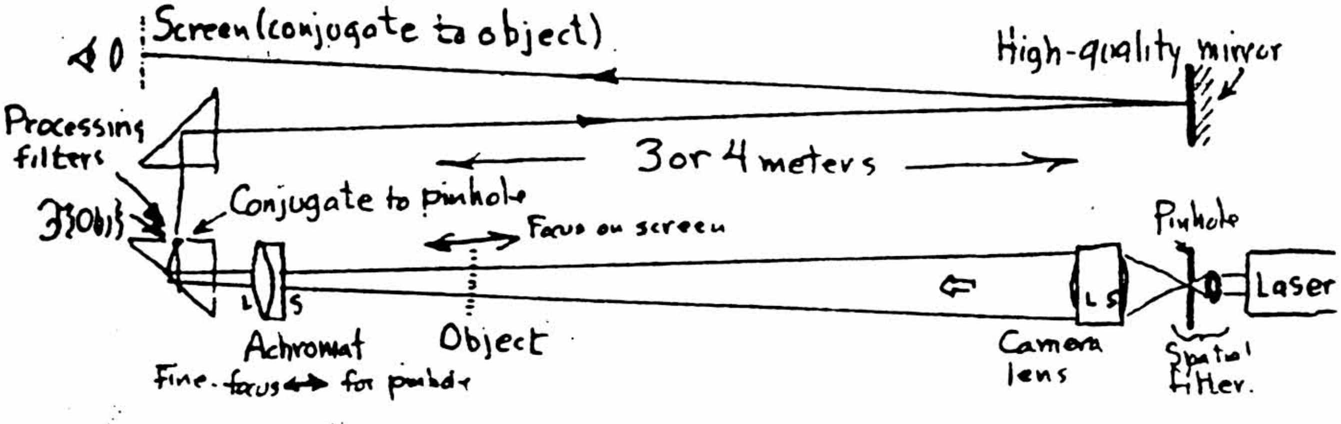
(b) The **brightness of the center spot** is a good clue. For example, a bunch of clear holes will pass a lot less total light than the complement -- a corresponding bunch of black dots. Except for the center spot, the patterns are indeed identical. Actually these (photographic) objects are far from perfect but they give you a pretty good feel for things.

(c) Highly **repetitious** objects yield repetitious **sharp** patterns.

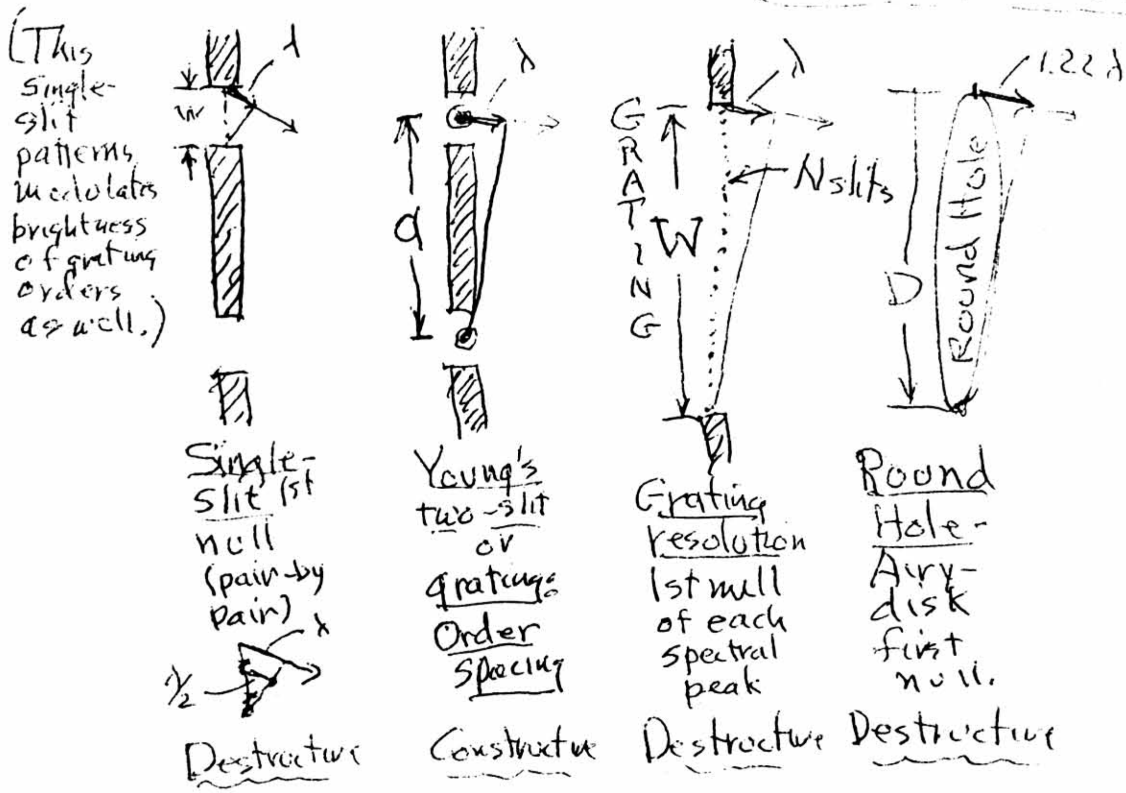
(d) When identical small objects are scattered about **randomly**, you can expect basically the same pattern as from just one of these small objects, but the random superposition of many will add their **intensities**.

(e) More of these pearls of wisdom could be put down, but giving too many might do more to confuse than to clarify.





Bread-and butter diffraction cases.



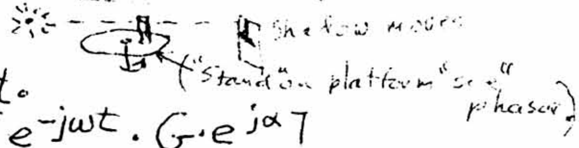
Quick road to Fourier Optics.

I - Simplifying assumptions.

(a) Scalar wave model. (b) Linearity. (c) Coherence.

II - Stationary representation with phasors

(a) Rotating-platform model.



(b) Complex-number equivalent.

$$G(t) = \Re_e\left[e^{-j\omega t} \cdot G \cdot e^{j\alpha}\right]$$

Shadow, Rotator, Ampl, Phase

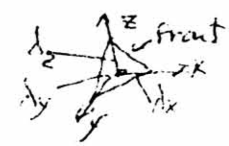
This part not written - "understood". phasor (= G too)

(c) Superposition. $G_1(t) + G_2(t) = (\text{phasor part } G_1 + G_2)$

III - Tilted plane wave

(a) Wave phase/ampl at origin is G

Direction of wave is $\hat{k}$



(b) Wave-number $= \sigma = 1/\lambda$ Call $\hat{k}\sigma \rightarrow (\sigma_x, \sigma_y, \sigma_z)$

(c) Information on one plane $z = z_0$, positions (x, y, z_0)

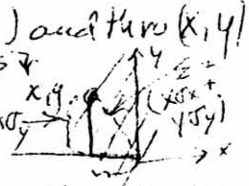
(d) Phase-difference between wavefront thru (0,0) and thru (x,y)

is $(x\sigma_x + y\sigma_y)\cdot 2\pi$ radians. Count wavefronts

Phasor for tilted wave at x,y is $G e^{j2\pi(x\sigma_x + y\sigma_y)}$ $N_0 = \frac{y}{\lambda_y} = y\sigma_y$

Thus $G(x, y, z_0, t) = \Re_e\left[e^{-j\omega t} \cdot e^{j\sigma_z z_0} \cdot G e^{j2\pi(x\sigma_x + y\sigma_y)}\right]$

All this now "understood" — Tilted wave at x,y



$N_0 = x/\lambda_x = x\sigma_x$

IV Superposition on x,y of flock of tilted plane waves.

(a) Continuous function $G(\sigma_x, \sigma_y)$ - contribution from $d\sigma_x\, d\sigma_y = G(\sigma_x, \sigma_y)\, d\sigma_x\, d\sigma_y$

(b) Superpose: $g(x,y) = \iint G(\sigma_x, \sigma_y)\, d\sigma_x\, d\sigma_y\, e^{j2\pi(x\sigma_x + y\sigma_y)} = \mathcal{F}^{-1}\{G(\sigma_x, \sigma_y)\}$

(Inverse) Fourier Transf.

V Reverse: From points $g(x,y)$ get Fraunhofer pattern at ∞

Superpose by Huygens - Freshel Principle.

$$G(\sigma_x, \sigma_y) = \iint g(x,y)\, dx\, dy\, e^{-j2\pi(x\sigma_x + y\sigma_y)} = \mathcal{F}\{g(x,y)\}$$

Fourier Transform.

Conclusion that Fourier Transforms drop right out of straightforward superposition.

The practical tools for Fourier Optics.

THEOREMS

NB → **Fourier Integral Theorem:** $\mathcal{F}\{\mathcal{F}^{-1}\{\underset{\sim}{u}\}\} = \underset{\sim}{u}$ and $\mathcal{F}^{-1}\{\mathcal{F}\{\underset{\sim}{u}\}\} = \underset{\sim}{u}$

NB → **Convolution Theorem:** $\mathcal{F}\{\underset{\sim}{g} \star \underset{\sim}{h}\} = \underset{\sim}{G} \cdot \underset{\sim}{H}$

Linearity: $\mathcal{F}\{\alpha \underset{\sim}{g} + \beta \underset{\sim}{h}\} = \alpha \underset{\sim}{G} + \beta \underset{\sim}{H}$

Similarity: $\mathcal{F}\{\underset{\sim}{g}(\alpha x, \beta y)\} = \underset{\sim}{G}(\sigma_x/\alpha, \sigma_y/\beta)$ [small ↔ big]

Also Shift 6/1-18 and Parseval 6/1-19

Cartesian-separable: $\underset{\sim}{G}(\sigma_x, \sigma_y) = \underset{\sim}{G}_x(\sigma_x) \cdot \underset{\sim}{G}_y(\sigma_y)$

Most we use are of this C.S. class. Not all, though

TRANSFORMS

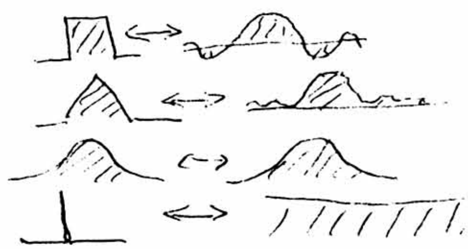
(a) Cartesian-separable, $\underset{\sim}{G}_x(\sigma_x)$ or $\underset{\sim}{G}_y(\sigma_y)$.

Rect ↔ sinc

Triangle ↔ sinc^2

Gauss ↔ Gauss

Delta ↔ Unity



Aperture is simply a wide rectangle ↔ sinc

Combinations important.

(b) Rotational symmetry:

Round hole ↔ Airy disk

Round Gauss ↔ Round Gauss

Fourier-Bessel Transform for general circular symmetry.

PROBLEMS (Due at start of Lecture 6/1)

6/1 − 1 ; 6/1 − 3 ; 6/1 − 5.

Note, Some "$\mathcal{F}$'s in text slipped thru as "QF"s

ANALYTICAL OUTLINE OF PATH FROM E.M. THEORY TO HUYGENS-FRESNEL PRINCIPLE.

① WAVE EQ'N IN PHASOR FORM:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \Rightarrow (\nabla^2 + k^2)\underset{\sim}{U}(\vec{P}) = 0$$

② **D.E. ⇒ $\iint_S$** Green's Theorem:

$$\iiint_V (\underset{\sim}{G}\nabla^2\underset{\sim}{U} - \underset{\sim}{U}\nabla^2\underset{\sim}{G})\,dv = \iint_S (\underset{\sim}{G}\underset{\sim}{U}_n - \underset{\sim}{U}\underset{\sim}{G}_n)\,ds$$

CHOOSE $\underset{\sim}{G}$ fcn sol'n same DE as $\underset{\sim}{U}$

$\underset{\sim}{G}$: wave expanding from $\vec{P}_j$ or

$\underset{\sim}{G} = \exp(j r_{ij})/r_{ij}$ abbrev. as **E_{ij}**

Integral Th. Kirchh. + Helmh.:

$$\underset{\sim}{U}(\vec{P}_0) = \frac{1}{4\pi}\iint_S \left(\underset{\sim}{E}_{01}\,\partial/\partial n - \partial\underset{\sim}{E}_{01}/\partial n\right)\underset{\sim}{U}\,ds$$

③ Imbed barrier, B, aperture, Σ, → limits.

Kirchhoff:

$$\underset{\sim}{U}(\vec{P}_0) = \frac{1}{4\pi}\iint_\Sigma E_{01}\left(\partial/\partial n - jk(\hat{n}\cdot\hat{r}_{01})\right)\underset{\sim}{U}\,ds$$

④ Illum. aperture from source at P_2.

(Also Rayleigh-Sommerfeld mirror-image $\underset{\sim}{G}$'s) **Eq's 6/3-9**

Reduce in practice to:

$$\underset{\sim}{U}(\vec{P}_0) = \iint_\Sigma (\underset{\sim}{A}\cdot\underset{\sim}{E}_{12})\cdot \underset{\sim}{E}_{01}\cdot\frac{1}{j\lambda}\cdot ds_\perp$$

↑ Ampl. at P_0; ↑ Super-position; ↑ Illum at aper.; ↑ Phase & ampl. at P_0; ↑ "fix-up factor"; ↑ Projected-area elements

THIS IS BASICALLY HUYGENS-FRESNEL PRINCIPLE.

① DIFFRACTION AS SUPERPOSITION INTEGRAL.

$$\underset{\sim}{U}(\vec{P}_0) = \iint_\Sigma [\underset{\sim}{U}(\vec{P}_1)\cdot \underset{\sim}{h}(\vec{P}_0,\vec{P}_1)]\,ds$$

where $\underset{\sim}{h}$ is eq'n ⑬.

Special case of Fraunhofer:

$$\underset{\sim}{U}(x_0/\lambda z, y_0/\lambda z) \propto \underset{\sim}{A}_0\, \mathcal{F}\{\vec{P}\}$$ (Eqn 14)

② FRESNEL APPROXIMATION:

Cartesian-separable cases → new Fresnel ∫.

$$\Rightarrow \underset{\sim}{U}\left(\frac{x_1}{M_e}, \frac{y_1}{M_e}\right) = \int_{x_1\,limit}^{x_1\,limit} \exp j(\pi/2)\xi^2 d\xi \cdot y\int \text{too}.$$

... and more ....

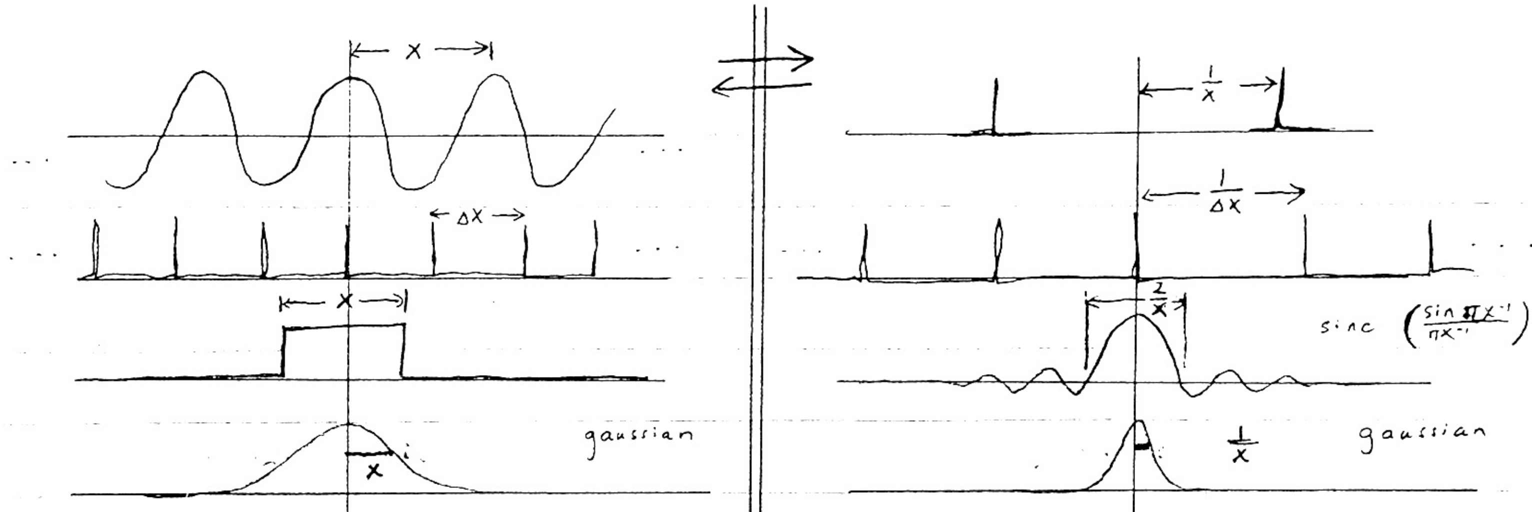
BABINET'S PRINCIPLE. Important.

Given a pt. that is dark but → bright when an opaque obstacle inserted. Replacing obstacle by its complement, the pt becomes exactly as bright as before.

Proof by Huygens-Fresnel principle is easy — the phase of light in the two cases at the point is opposite. Hence cancels and → dark with no obstacle.

PROBLEMS for Fri 94/4/22 6/4-1 | Optional 6/4-2.

Basic Fourier Transforms



Normal FT $\propto N^2$
FFT $\propto N \ln N$

$g(x) \rightleftarrows G(\sigma)$

$h(x) \rightleftarrows H(\sigma)$

Convolution Theorum:

$g, h \rightleftarrows G, H$

(Convolution $= N^2$ multiply + adds) $\circledast \downarrow$ $\downarrow \cdot$ (Product : $= N$ multiplies)

$g \circledast h \rightleftarrows G \cdot H$

Also:

$g, h \rightleftarrows G, H$

(product) $\cdot \downarrow$ $\downarrow \circledast$ (convolution)

$g \cdot h \rightleftarrows G \circledast H$

Useful Extras:

Similarity : $g(ax) \rightleftarrows G\left(\frac{\sigma}{a}\right)$ (If $g(x) \rightleftarrows G(\sigma)$)

Shift : $g(x-a) = G(\sigma)\, e^{i \cdot 2\pi a \sigma_x}$

g real $\Rightarrow G(-x) = G^*(x)$ $\therefore$ Both real $\Rightarrow$ Both symmetrical

($* \Rightarrow$ change sign of imaginary part)

Parseval : $\int_{-\infty}^{\infty} g^* g \, dx = \int_{-\infty}^{\infty} G^* G \, d\sigma$