Mutual Impedance of Receiving Array and Calibration Matrix for High-resolution DOA Estimation

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1 Introduction

Precise array calibration is necessary to realize high-resolution capability of superresolution techniques such as the MUSIC algorithm[1]. Elements imbalance and mutual coupling among the elements of the array should be calibrated to realize high-resolution DOA (Direction-of-Arrival) estimation. Many researches have been done for the calibration[2]-[4]. Among them, calibration of the mutual coupling is the difficult problem. For the array of single mode elements, the calibration matrix becomes a constant scan-independent matrix. Gupta and Ksienski shown that the mutual coupling can be expressed by the function of mutual impedance. However, the matrix is not perfect. Hui proposed a redefined mutual impedance matrix to improve the performance[3]. Adve and Sarkar derive a new method for the exact mutual coupling matrix[4]. The method can calculate precise coupling matrix, however, it requires numerical calculation of the antenna model. A calibration method which can be done only with measured values, such as S parameters is desired for practical applications. In this report, we show the problems of the conventional equivalent circuit representation of an array, and propose a new definition of mutual impedance. Validity of the proposed calibration method is evaluated numerically by using the DOA estimation results of the MUSIC spectrum.

2 Array Calibration in DOA Estimation

In this report, we consider 1-D DOA estimation problem with a 1-N element uniform linear array (ULA) as shown in Fig.1. Assuming that there exist d incident plane waves incoming from $\theta_i(i = 1 \sim d)$ whose amplitude is s_i. In this case, a received data vector of the array can be given by

$$\boldsymbol{r} = [r_1, r_2, \cdots, r_N]^T = \boldsymbol{CAs} + \boldsymbol{n} \tag{1}$$

where ^{T} denotes transpose and $\boldsymbol{n}$ is a additive noise having power of σ^2. The matrix $\boldsymbol{A}$ is a $N \times d$matrix whose columns are the mode vectors $(\boldsymbol{a}(\theta_i))$. $\boldsymbol{s}$ is a $d \times 1$ signal vector. The $N \times N$ matrix $\boldsymbol{C}$ is a mutual coupling matrix whose (i, j) elements show the coupling coefficients between i-th and j-th elements. The array having scan-independent matrix $\boldsymbol{C}$ is considered in the following, that is *single-mode* array.

When the $\boldsymbol{C}$ is known, a covariance matrix of the data, $\boldsymbol{R}$ can be calibrated by

$$\boldsymbol{R}_{\rm cal} = \boldsymbol{C}^{-1}(\boldsymbol{R} - \sigma^2\boldsymbol{I})(\boldsymbol{C}^H)^{-1}. \tag{2}$$

where ^{H} denote complex conjugate transpose. The matrix can be widely applied for high-resolution DOA estimation techniques. For the MUSIC algorithm[1], we can also calibrate the spectrum as follows:

$$P_{\rm music}(\theta) = \frac{\boldsymbol{a}(\theta)^H(\boldsymbol{C}^H\boldsymbol{C})\boldsymbol{a}(\theta)}{\boldsymbol{a}(\theta)^H(\boldsymbol{C}^H\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{C})\boldsymbol{a}(\theta)}, \tag{3}$$

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where $\boldsymbol{E}_n$ denotes noise-subspace matrix[1]. The problem is how to estimate the matrix $\boldsymbol{C}$. Gupta *et. al.* derived by using equivalent circuit of array that the coupling matrix array becomes[2],

$$\boldsymbol{C}^{-1} = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \cdots & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & & \frac{Z_{2N}}{Z_L} \\ \vdots & & \ddots & \vdots \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \cdots & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix} = \boldsymbol{I} + \frac{1}{Z_L}\boldsymbol{Z}_m, \tag{4}$$

where Z_L is a known load impedance, and Z_{ij} represents the mutual impedance of ports i and j. The matrix works good for a single mode array, however, as the several researchers reported, above matrix is not a precise mutual coupling matrix[3],[4].

3 Mutual Impedance of Arrays

Recently, several researcher have been reported the problem of equivalent circuit for antennas[5]. These reports show that the Thèvenin equivalent circuit cannot correct for the antennas including reradiated and scattered phenomena. This effects the mutual impedance calculation of the array. Here, we examine the mutual impedance of 2-el dipole array numerically. The array parameters are listed in Table.1. We use the method of moments (NEC2) in this calculation.

When #1 port is excited by V_g, we can express the equivalent circuit as shown in Fig.2. Port currents (i_1, i_2) and antenna terminal voltages (v_1', v_2') are calculated by the NEC2. Here, we assume that the Z_{ii} is known. We can easily calculate Z_{ii} for the model of single element numerically. In this case, $Z_{11} = Z_{22} = 72.7 + j2.52(\Omega)$. Using these values, the mutual impedances can be estimated as follows:

$$Z_{12} = \left(\frac{v_1'}{i_1} - Z_{11}\right)\frac{i_1}{i_2} = -10.1 - j31.6 \quad (\Omega) \tag{5a}$$

$$Z_{21} = \left(\frac{v_2'}{i_2} - Z_{22}\right)\frac{i_2}{i_1} = -14.1 - j30.0 \quad (\Omega) \tag{5b}$$

Clearly, these mutual impedances are different. Z_{21} relates to the transmission from #1-element to #2-element. On the other hand, Z_{12} relates to the reradiation from #2 to #1. In this report, we define Z_{21} and Z_{12} is the transmission and the reradiation mutual impedance, respectively. In the following considerations, we use the notation Z_{ij}^s for the reradiation mutual impedance.

4 Calibration Matrix for Receiving Arrays

Here, we consider the mutual coupling problem of N-el array by using the proposed definition. Relation between voltages and current of each port can be written by

$$\begin{bmatrix} v_0' \\ v_1' \\ \vdots \\ v_N' \end{bmatrix} = \begin{bmatrix} V_{g0} - Z_L i_0 \\ -Z_L i_1 \\ \vdots \\ -Z_L i_N \end{bmatrix} = \begin{bmatrix} Z_{00} & Z_{01}^s & Z_{02}^s & \cdots & Z_{0N}^s \\ Z_{10} & Z_{11} & Z_{12}^s & \cdots & Z_{1N}^s \\ Z_{20} & Z_{21}^s & Z_{22} & \cdots & Z_{2N}^s \\ \vdots & \vdots & & \ddots & \vdots \\ Z_{N0} & Z_{N1}^s & Z_{N2}^s & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_0 \\ i_1 \\ \vdots \\ i_N \end{bmatrix} \tag{6}$$

where #0 denotes transmitting ports and #1∼#N represent ports of the receiving array. Since $-Z_{j0}i_j (j = 1 \sim N)$ is the (uncoupled) open circuit voltage of j-th port of the receiving array, we can derive relation between open circuit voltages($\boldsymbol{v}_{\text{open}}$) and (coupled) terminal voltages($\boldsymbol{v}$):

$$\boldsymbol{v}_{\text{open}} = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z^s_{12}}{Z_L} & \cdots & \frac{Z^s_{1N}}{Z_L} \\ \frac{Z^s_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & & \frac{Z_{2N^s}}{Z_L} \\ \vdots & & \ddots & \vdots \\ \frac{Z^s_{N1}}{Z_L} & \frac{Z^s_{N2}}{Z_L} & \cdots & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \left(\boldsymbol{I} + \frac{1}{Z_L}\boldsymbol{Z}_s\right)\boldsymbol{v} = \boldsymbol{C}^{-1}\boldsymbol{v} \quad (7)$$

This equation shows that *reradiation mutual impedance* should be used for calibration. The impedances can be estimated by the receiving array alone. When each port is excited by V_g separately, we can obtain the following equation:

$$\begin{aligned} & diag\{V_g, \cdots, V_g\} = \\ = \quad & (\boldsymbol{Z}_m + Z_L\boldsymbol{I}) \begin{bmatrix} i_{11} & & \text{\Large 0} \\ & \ddots & \\ \text{\Large 0} & & i_{NN} \end{bmatrix} + (\boldsymbol{Z}_s + Z_L\boldsymbol{I}) \begin{bmatrix} 0 & i_{12} & \cdots & i_{1N} \\ i_{21} & 0 & & i_{2N} \\ \vdots & & \ddots & \vdots \\ i_{N1} & i_{N2} & \cdots & 0 \end{bmatrix} \end{aligned} \quad (8)$$

This is a N^2 simultaneous equation having N^2 unknowns $(Z_{ii}, Z_{ij}, Z^s_{ij})$. For an ULA, only $\left\lfloor \frac{N^2}{2} \right\rfloor$ equations are independent because of the array geometry. However, number of unknowns also becomes $2N-1$ since $\boldsymbol{Z}_m$ and $\boldsymbol{Z}_s$ become Toeplits matrices. Therefore, all mutual impedances can be estimated for $N > 2$.

5 Numerical Results

In this section, we shown the calibration performance by using the MUSIC spectrum. As we shown in the sect.2, calibrated MUSIC spectrum can be plotted by (3). The MUSIC spectrum, $P_{\text{music}}(\theta)$ becomes infinity for the corresponding DOA(s) of incoming wave(s) when the data covariance can be estimated correctly (SNR$\rightarrow\infty$ or infinite number of snapshots) with precise calibration matrix $\boldsymbol{C}$. In the followings, we assume that SNR is infinity (no noise), and the received data can be calculated by using the NEC2.

Figure 3 shows the calibrated and uncalibrated MUSIC spectrums of 4 data-sets. Each data-set have one incident waves from $0°, 20°, 45°, 60°$, respectively. The spectrums denoted "raw data" show the uncalibrated spectrums. "conventional-cal." and "proposed-cal." corresponds to the spectrum calibrated by the Hui's rede-fined impedance matrix[3] and the proposed calibration matrix shown in Sec.4, respectively. Since $N = 2$ in this case, we assume that Z_{ii} is known and use the reradiated mutual impedance value shown in Sec.2. As can be seen in this figure, bias can be removed and sharp peaks can be obtained by the proposed method. Figures 4(a) and (b) show the MUSIC spectrums for 4-el array with $Z_L = 50\Omega$ and 100Ω, respectively. Other parameters are the same in Table.1. In these cases, we can see that the bias and peaks can be improved by the proposed calibration method.

6 Conclusions

In this report, we propose a new calibration matrix by using the *re-radiation and transmission* mutual impedances. The numerical results show that accuracy of the calibration matrix can be improved by our definition. By using the method, mutual coupling effect of single-mode arrays can be calibrated with measured S parameters of the receiving array without external reference signals and/or numerical calculation of current distributions. Therefore, the proposed method would be useful for high-resolution DOA estimation with dipole or monopole arrays.

References

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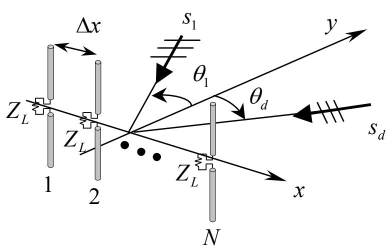


Fig.1 DOA estimation with N-element array.

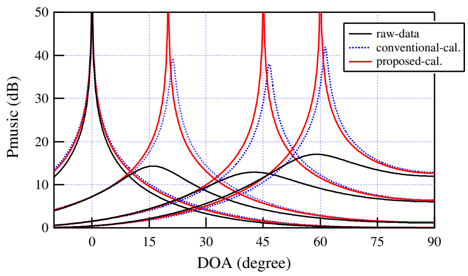
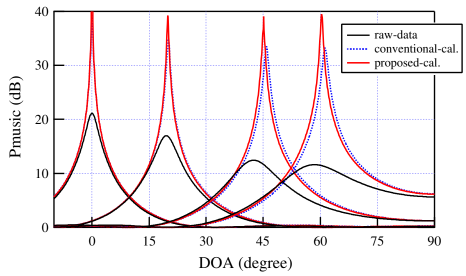


Fig.3 MUSIC spectrums with the 2-element dipole array.

Table.1 Configulation of the array.

Frequency	2.4 GHz
Length of wire	5.8 cm (0.464 λ)
Radius of wire	0.5 mm
Load impedance	50 Ω
Element separation	6 cm (0.48 λ)



(a)$Z_L = 50\Omega$

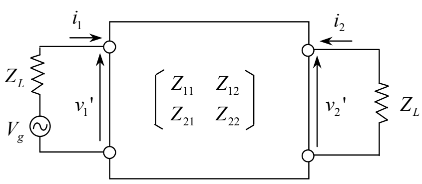
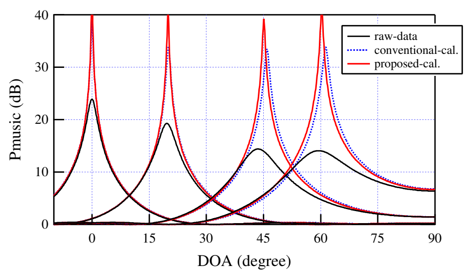


Fig.2 Equivalent cuircuit of the 2-el. array.



(b)$Z_L = 100\Omega$

Fig.4 MUSIC spectrums with the 4-element dipole array.